IB Math Studies – Review Packet 1

Each question is worth 6 marks. You do not have to show work to earn full marks.

1. Given $f(x) = \frac{5}{x^3} - \frac{2}{x^2} + \frac{3}{x} - 6x + 1$,
   
   (a) Calculate $f'(x)$.
   
   (b) Find $f'(-1)$.
   
   (c) Explain what the value of $f'(-1)$ represents.

2. Let $p$ stand for the proposition “I will wear a hat.” Let $q$ stand for the proposition “it is cloudy.”

   (a) Write the following statements in symbolic logic form:
      
      (i) “I will wear a hat is and only if it is not cloudy.”
      
      (ii) “Either I will not wear a hat or it will be cloudy, but not both.”

   (b) Write down, in words, the contrapositive of the statement, “If it is cloudy, then I will wear a hat.”

3. The table below shows the number of men and women in a small town who voted in a local election.

<table>
<thead>
<tr>
<th></th>
<th>Voted</th>
<th>Did not vote</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>57</td>
<td>10</td>
<td>67</td>
</tr>
<tr>
<td>Female</td>
<td>45</td>
<td>18</td>
<td>63</td>
</tr>
<tr>
<td>Total</td>
<td>102</td>
<td>28</td>
<td>130</td>
</tr>
</tbody>
</table>

   (a) If a person was selected at random from the town, find the probability that the person
      
      (i) voted and is female,
      
      (ii) did not vote,
      
      (iii) did not vote, given that the person selected is male.

   (b) If two randomly selected people were selected from the town, find the probability that both voted.
4. A bag contains 6 red and 4 green candies.

Pauline randomly selects one sweet out of the bag and eats it. Then she randomly selects a second sweet. Below is a tree diagram showing Paulina’s possible choices. Two of the probability values are missing.

(a) Fill in the missing probability values on the tree diagram.

(b) If Paulina eats two candies, what is the probability that she will eat at least one green?

(c) What is the probability Paulina will select a red candy given the first was green?

5. Paige is training for an endurance cycling challenge. She rides 1.5 kilometers in her first week of training, 2.25 kilometers during the second week, 3 kilometers during the third, and so on.

(a) Calculate the number of kilometers Paige rides during her tenth week of training.

(b) Find the total number of kilometers ridden by the end of her tenth week of training.

The cycling challenge involves riding 125 kilometers in one day.

(c) Determine the number of training weeks Paige must ride before she has ridden a total of 125 kilometers.

6. A professor surveyed 200 recent college graduates to determine if the degree obtained was independent of employment status. The majors were engineering, education, marketing, accounting, and computer science. The graduates were either employed or unemployed. A $\chi^2$ test was conducted at the 5% significance level.

(a) Write down the null hypothesis.

(b) Find the number of degrees of freedom for this test.

(c) If the calculated $p$-value was 0.032, determine if the degree obtained is independent of employment status.

Give a clear reason for your answer.
7. The data below display the number of cars parked at a shopping center on 21 randomly selected days last month.

<table>
<thead>
<tr>
<th></th>
<th>5</th>
<th>8</th>
<th>13</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>28</td>
<td>31</td>
<td>33</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>40</td>
<td>41</td>
<td>41</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Determine the
   (i) lower quartile,
   (ii) median,
   (iii) upper quartile.

(b) Complete the frequency table below.

<table>
<thead>
<tr>
<th>Number of Cars</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 − 10</td>
<td></td>
</tr>
<tr>
<td>11 − 20</td>
<td></td>
</tr>
<tr>
<td>21 − 30</td>
<td></td>
</tr>
<tr>
<td>31 − 40</td>
<td></td>
</tr>
<tr>
<td>41 − 50</td>
<td></td>
</tr>
</tbody>
</table>

(c) State whether the data is discrete or continuous.

8. Let $X$ be normally distributed with a mean of 75 and a standard deviation of 10.

(a) On the diagram below, shade the area representing $P(X < 65)$.

(b) Calculate the area of the shaded region above.

(c) Find $P(65 < X < 95)$. 
9. A boat is 450 meters from the base of a cliff. The angle of elevation from the boat to the top of the cliff is 25°.
   (a) Draw a diagram representing the situation. Clearly label the distance and the angle given.
   (b) Find the height of the cliff in kilometers.

The boat moves closer to the base of the cliff such that the angle of elevation increases to 30°.
(c) Determine the distance traveled by the boat.

10. Ariana was in charge of a game for children at a school festival. Children tossed a bean bag from a set distance and tried to land the bean bag in a small basket. They could toss the bean bag as many ties as they needed in order to win a piece of candy.
    During a one-hour period, she recorded the number of tosses it took each child before winning.
    The results are shown in the frequency table below.

    | Number of Tosses | Frequency |
    |-----------------|-----------|
    | 1 – 3           | 3         |
    | 4 – 6           | 6         |
    | 7 – 9           | 9         |
    | 10 – 12         | 5         |
    | 13 – 15         | 2         |

(a) Calculate an approximate
   (i) mean,
   (ii) standard deviation.
(b) What is the probability a child would need more than nine tosses to win?
(c) Suppose the number of tosses follows a normal distribution. What is the probability a randomly selected child would need fewer than five tosses?