Each question is worth 6 marks. You do not have to show work to earn full marks.

1. An ice-cream cone with one scoop of ice cream, as shown below, has a vertical angle of 20° and a slant height of 15 cm.

(a) Calculate the length of the radius of the ice-cream cone.

(b) Calculate the volume of the cone with the scoop of ice cream. Round the answer to the nearest cubic centimeter.

2. The diagram below shows the graphs of two functions, \( f(x) = 4x^2 - 1 \) and \( g(x) = 2^x \).

(a) Write down the coordinates of \( B \), the \( y \)-intercept of \( f(x) = 4x^2 - 1 \).

(b) Determine the \( x \)-coordinates for the two points, \( A \) and \( C \), where the graphs intersect.

(c) Determine the values of \( x \) for which \( 4x^2 - 1 < 2^x \).
3. Genghis invests 3 100 Chinese Yuan in an account that earns 3.25% interest per annum compounded quarterly.
   (a) Calculate, to the nearest Yuan, how much Genghis will have in his account after 5 years.

When Genghis has saved 3 800 Yuan, he will vacation in Italy.
(b) Determine the number of years before Genghis will have saved enough money for his vacation.
(c) Before he leaves, Genghis converts 2 000 Yuan to euro for spending money. If the exchange rate is 1 Yuan = 0.12 Euro, how many euros will Genghis receive?

4. Consider the function \( g(x) = \frac{1}{2}x^2 - 2 + \frac{4}{x} \).
   (a) On the grid below, sketch the graph of \( g(x) \) for the domain \(-4 \leq x \leq 5\).

   (b) Write down the value of \( x \)-intercept to the nearest tenth.

   (c) On the same grid, sketch the graph of \( h(x) = 2^{-x} + 5 \) for the domain \(-4 \leq x \leq 5\).

   (d) Hence or otherwise, solve \( g(x) = h(x) \).
5. The truth table shows part of the truth values for the compound statement \([\neg q \land (p \Rightarrow q)] \Rightarrow \neg p\)

<p>| | | | | | | | |</p>
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<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>\neg p</td>
<td>p \Rightarrow q</td>
<td>\neg q \land (p \Rightarrow q)</td>
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<td>\neg q \land (p \Rightarrow q) \Rightarrow \neg p</td>
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(a) Fill in the missing truth values on the table.

(b) Determine if the argument \([\neg q \land (p \Rightarrow q)] \Rightarrow \neg p\) is logically valid.

**Question #6 is worth a varied amount of marks for each part. You must show ALL work to earn full marks.**

6. Below are the scores for 15 students on their final examination for twelfth grade math and English.

<table>
<thead>
<tr>
<th>Math (x)</th>
<th>68</th>
<th>88</th>
<th>92</th>
<th>55</th>
<th>72</th>
<th>79</th>
<th>81</th>
<th>64</th>
<th>32</th>
<th>95</th>
<th>71</th>
<th>82</th>
<th>86</th>
<th>61</th>
<th>78</th>
</tr>
</thead>
<tbody>
<tr>
<td>English (y)</td>
<td>72</td>
<td>86</td>
<td>92</td>
<td>65</td>
<td>68</td>
<td>85</td>
<td>85</td>
<td>70</td>
<td>43</td>
<td>99</td>
<td>79</td>
<td>80</td>
<td>82</td>
<td>70</td>
<td>78</td>
</tr>
</tbody>
</table>

(a) Draw a scatter diagram to show the data above. Let 1 cm represent 10 on both the horizontal and vertical axes.

(b) Write down the Pearson’s product-moment correlation coefficient, \(r\), for the data.

(c) Describe what the correlation coefficient suggests about the relationship between the two variables.

(d) Calculate the mean point.

(e) Write the equation of the regression line for \(y\) on \(x\) un the form \(y = ax + b\).

(f) Draw the regression line on the scatter diagram.

(g) Use your equation for the regression line to predict the math score for a student who scored a 75 on the English examination.

(h) Determine if the prediction from part (g) is inappropriate. Justify your answer.

(Total 17 marks)

Each question is worth a different amount of marks. You must show ALL work to earn full marks.

1. A sheet of square cardboard with side length 12 cm will be made into an open box by cutting equal-sized squares from each corner and folding up the four edges, as shown below.

(a) Write an expression in terms of $x$ for the side length of the box. (1)

(b) Show that the volume, $V(x)$, of the box will be $V(x) = 4x^3 - 48x^2 + 144x$. (2)

(c) Find $V'(x)$. (2)

(d) Find the height of the box that yields the greatest volume. (3)

(e) Determine the maximum volume possible. (2)

A sheet of cardstock is to be rolled such that a cylinder is formed. The cylinder has the same volume as the open-top box and the height of the cylinder is 8 cm.

(f) Determine the radius of the cylinder. Round your answer to the nearest centimeter. (4)

(Total 14 marks)
2. Sixty high school students are asked of which of the following academic clubs they are a member: Science Club (S), Math Club (M), or History Club (H).

- 28 students are members of the Science Club
- 28 students are members of the Math Club
- 26 students are members of the History Club
- 14 students are members of the Science and the Math Club
- 11 students are members of the Science and the History Club
- 10 students are members of the Math and the History Club
- 1 student is a member of all three

(a) Represent the above information on a Venn diagram. Clearly label each region with the number of students. (4)

(b) Find the number of students that are not members of any of the academic clubs. (2)

(c) A student is chosen at random. Find the probability that the student is a member of:
   (i) the History Club or the Math Club, but not both;
   (ii) the Science Club, given the student is a member of the Math Club. (4)

The students were also asked to identify their postsecondary education plans. The results are displayed below and categorized based on gender.

<table>
<thead>
<tr>
<th>Post Secondary Plans</th>
<th>Attend 2 year college</th>
<th>Attend 4 year college</th>
<th>Begin Working</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>10</td>
<td>14</td>
<td>9</td>
<td>32</td>
</tr>
<tr>
<td>Female</td>
<td>14</td>
<td>9</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>23</td>
<td>13</td>
<td>60</td>
</tr>
</tbody>
</table>

A chi-squared test of independence is to be performed to determine if there is any association in postsecondary plans and gender.

(d) State the null hypothesis. (1)

(e) Show that the number of degrees of freedom is 2. (1)

(f) Write down the:
   (i) chi-squared calculated value,
   (ii) $p$-value. (2)
At the 5% significance level $\chi^2 = 7.81$.

(g) Determine the conclusion of the chi-squared test. Justify your answer. (2)  

(Total 16 marks)

3. $V ABCD$ is a square-based pyramid, and $V$ is directly above the center of the square.

(a) Find:
   (i) the length $AM$,
   (ii) the height $VM$.  

(4)

(b) Calculate the size of the angle $VAM$.  

(2)

(c) Calculate the height of one of the pyramid’s triangular faces.  

(2)

(d) Calculate the total surface area of the pyramid.  

(3)

A square cuboid is to be made with the same surface area as the pyramid.

(e) Determine the length of the side, $s$, of the cuboid.  

(3)  

(Total 14 marks)
4. Thomas has a custom furniture business, and he is planning to launch a website to market his products worldwide.

In 2010, he invested 10 000 USD in a savings account that earns 4.5% interest per annum compounded monthly.

(a) How much interest will Thomas have earned after 5 years? Give your answer to the nearest dollar. (3)

Thomas needs 15 000 USD to launch his website.

(b) Determine the number of years before Thomas will have enough money to launch his website. (3)

In 2010 Thomas purchased a table saw for 1 200 USD, but the table saw depreciates at a rate of 9.8% per annum.

(c) Find the value of the table saw after 5 years. Round answer to the nearest hundredth. (2)

The sofa table is the most popular piece of furniture ordered by customers. Thomas sells the sofa table for 350 USD plus an 8% shipping and handling fee.

(d) Determine the cost a customer pays for one sofa table. (2)

(e) A customer in Italy purchases a sofa table. Find her cost in euro if 1 euro = 1.30 USD.

   Round answer to the nearest euro. (2)

The monthly profit Thomas earns from the sales of the sofa table follows the function $P(x) = -2x^2 + 120x - 75$.

(f) Calculate the maximum profit Thomas can earn. (2)

(Total 14 marks)
5. The quadrilateral $ABCD$ graphed below has vertices at $A(-3, 3), \ B(0, 6), \ C(2, 2),$ and $D(-2, 1)$.

(a) Calculate the gradient of line $AD$.  \hspace{1cm} (2)

(b) Show that line $AD$ is parallel to line $BC$.  \hspace{1cm} (2)

(c) Write the equation of line $AB$ in the form $ax + by + d = 0$, where $a, b, c \in \mathbb{Z}$.  \hspace{1cm} (2)

The line $AB$ intersects line $CD$ at point $P$.

(d) Determine the coordinates of point $P$.  \hspace{1cm} (2)

Consider the triangle $BCP$.

(e) Calculate the length of side:

(i) $BP$,

(ii) $CP$.  \hspace{1cm} (4)

The length of side $BC$ is $\sqrt{20}$.

(f) Calculate the size of angle $BPC$.  \hspace{1cm} (3)

(Total 15 marks)
1. The diagram shows a part of the curve $y = f(x)$.

(a) Write down the values of $x$, when $f'(x) = 0$.

(b) Write down the interval of $x$ for which $f'(x) > 0$.

(c) Determine if the gradient of the tangent line at $x = -3$ will be positive or negative. Justify your answer.

2. The third term of a geometric sequence is 56, and the sixth term is 7.

(a) Determine the value of the common ratio.

(b) Calculate the value of the first term.

(c) Find the sum of the geometric sequence for the first 10 terms.
3. Rachel recorded the number of students, \( t \), late to school each day for an entire month. She displayed the results in the histogram below. She used the intervals \( 0 \leq t < 2, 2 \leq t < 4 \), etc.

![Histogram](image)

(a) Write down the modal class.

(b) Write down an estimate standard deviation using the graphics display calculator.

(c) Find the probability that at least 6 students would be late to school on a randomly selected school day.

4. Garrett is designing a small storage shed; the sketch is shown below. The base of the shed will be 3 meters long and the total height will be 2.4 meters. Each side of the roof has a small overhang of 11 cm. The roof makes a 26.5° angle with the shed.

![Sketch](image)

(a) Calculate the height of the roof, \( r \), in meters.

(b) Determine the area of the triangular roof.

Garrett is going to paint the front rectangular face of the shed light grey before installing a door.

(c) Determine the number of square meters he will paint light grey.
5. Consider each of the following statements:

- \( p \): Sam passes math class
- \( q \): Sam passes history
- \( r \): Sam studies each night

(a) Write the following compound statement in words.

\[ \neg(p \lor q) \implies \neg r \]

(b) Complete the truth table for the compound statement in part (a).

\[
\begin{array}{cccccccc}
 p & q & r & \neg r & p \lor q & \neg(p\lor q) & \neg(p\lor q) \implies \neg r \\
 T & T & T & F & T & T & T \\
 T & T & F & T & T & T & T \\
 T & F & T & F & F & T & T \\
 T & F & F & T & F & T & T \\
 F & T & T & F & T & T & T \\
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 F & F & T & F & T & T & T \\
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\end{array}
\]

(c) Write down one example of when \( \neg(p \lor q) \implies \neg r \) is false.

6. The figure below is a triangular prism, where \( QT = 10 \text{ cm} \), \( QW = 7.5 \text{ cm} \), and \( ST = 18 \text{ cm} \).

\[
\begin{array}{cccccccc}
 p & q & r & \neg r & p \lor q & \neg(p\lor q) & \neg(p\lor q) \implies \neg r \\
 T & T & T & F & T & T & T \\
 T & T & F & T & T & T & T \\
 T & F & T & F & F & T & T \\
 T & F & F & T & F & T & T \\
 F & T & T & F & T & T & T \\
 F & T & F & T & F & T & T \\
 F & F & T & F & F & T & T \\
 F & F & F & T & F & T & T \\
\end{array}
\]

(a) Calculate the length of \( TW \).

(b) Determine the size of the angle between \( TW \) and the base.

(c) Find the surface area for the prism.
7. Consider the function \( f(x) = x^3 - x^2 \).
   (a) Write down \( f'(x) \).
   
   When \( x = a \), the gradient of the tangent line is 8.
   (b) Determine if \( f(x) \) is increasing or decreasing at \( x = a \). Justify your answer.
   (c) Calculate the value of \( a \) if \( a \in \mathbb{Z} \).

8. Devin invested 1 500 euro into an account with 2.5% nominal annual interest rate compounded quarterly.
   (a) Calculate the value of her investment after 10 years. Round answer to the nearest euro.
   
   After those 10 years, Devin's job relocates her to Japan. Her new bank will convert her investment to Japanese yen (JPY) but charges a 3% transaction fee.
   (b) Determine the amount of euro Devin will lose when switching to the Japanese bank. Round answer to the nearest hundredth.
   (c) Calculate to the nearest yen the amount Devin will have in her new account if the conversion rate is 1 euro = 130 JPY.

9. \( A \) and \( B \) are subsets of a universal set \( U \).
   \[ U = \{ x : x \in \mathbb{N}, 1 \leq x < 13 \}, A = \{ \text{multiples of 3} \}, \text{and } B = \{ \text{factors of 24} \} \]
   (a) List the elements of sets:
      (i) \( A \)
      (ii) \( A \cup B \)
      (iii) \( A' \cap B \)
   (b) Represent sets \( U \), \( A \), and \( B \) on a Venn diagram.