STANDARD FORM FOR COSINE: \( y = a \cos b (x - c) + d \)

The parent function of cosine is \( y = \cos x \)

1. In the parent function, \( a = 1 \), \( b = 1 \), \( c = 0 \), and \( d = 0 \). This means that:
   a) The amplitude is 1
   b) The period is \( \frac{2\pi}{b} = 2\pi \)

2. The significant points are \((\text{angle, } x\text{-coordinate of the Quadrantal})\):
   \((0, 1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 0)\)

3. The graph of the cosine parent function is:

TRANSFORMATIONS OF THE PARENT FUNCTION OF COSINE:

EXAMPLE 1: \( y = \cos x - 2 \)

Transformation(s): Vertical Shift down 2

Period: \( 2\pi \)

Amplitude: 1 \hspace{1cm} Domain: \([0, 2\pi]\) \hspace{1cm} Range: \([-3, -1]\)

Significant points:

\((0, -1), \left(\frac{\pi}{2}, -2\right), (\pi, -3), \left(\frac{3\pi}{2}, -2\right), (2\pi, -1)\)

\begin{align*}
(0, 1) & \quad \left(\frac{\pi}{2}, 0\right) & \quad (\pi, -1) & \quad \left(\frac{3\pi}{2}, 0\right) & \quad (2\pi, 1) \\
-2 & \quad -2 & \quad -2 & \quad -2 & \quad -2 \\
(0, -1) & \quad \left(\frac{\pi}{2}, -2\right) & \quad (\pi, -3) & \quad \left(\frac{3\pi}{2}, -2\right) & \quad (2\pi, -1) \\
\end{align*}
**EXAMPLE 2:** \( g(x) = 2 \cos \frac{1}{2} \left( x + \frac{\pi}{2} \right) \)

Transformation(s): **Vertical Dilation stretched by a factor of 2**,  
**Horizontal Dilation stretched by a factor of 2**,  
**Horizontal Shift left \( \frac{\pi}{2} \)**

**Period:** \( \frac{2\pi}{b} \)  
\( b = \frac{1}{2} \)  
therefore,  
\( \frac{2\pi}{\frac{1}{2}} = 2\pi \times \frac{1}{2} \)  
\( 2\pi \times \frac{2}{1} = 4\pi \)

**Period:** \( 4\pi \)  
**Amplitude:** 2  
**Domain:** \([-\frac{\pi}{2}, \frac{7\pi}{2}]\)  
**Range:** \([-2, 2]\)

**Significant points:**  
\( (0, 1) \) \( \left( \frac{\pi}{2}, 0 \right) \) \( (\pi, -1) \) \( \left( \frac{3\pi}{2}, 0 \right) \) \( (2\pi, 1) \)  
\( *2 \) \( *2 \) \( *2 \) \( *2 \) \( *2 \)

\( (0, 2) \) \( \left( \frac{\pi}{2}, 0 \right) \) \( (\pi, -2) \) \( \left( \frac{3\pi}{2}, 0 \right) \) \( (2\pi, 2) \)  
\( *2 \) \( *2 \) \( *2 \) \( *2 \) \( *2 \)

\( (0, 2) \) \( (\pi, 0) \) \( (2\pi, -2) \) \( (3\pi, 0) \) \( (4\pi, 2) \)  
\( -\frac{\pi}{2} \) \( -\frac{\pi}{2} \) \( -\frac{\pi}{2} \) \( -\frac{\pi}{2} \) \( -\frac{\pi}{2} \)

\( (-\frac{\pi}{2}, 2) \) \( \left( \frac{\pi}{2}, 0 \right) \) \( \left( \frac{3\pi}{2}, -2 \right) \) \( \left( \frac{5\pi}{2}, 0 \right) \) \( \left( \frac{7\pi}{2}, 2 \right) \)
STANDARD FORM FOR TANGENT: \( y = a \tan b(x - c) + d \)

The parent function of tangent is \( y = \tan x \)

The graph of tangent **DOES NOT** have amplitude because there is no minimum or maximum.

The period for tangent is \( \pi \). Graph tangent functions from \( -\frac{\pi}{2} \) to \( \frac{\pi}{2} \) on the unit circle.

Significant Points:
- \( \left( -\frac{\pi}{2}, a \right) \), \( \left( -\frac{\pi}{4}, -1 \right) \), \( (0, 0) \), \( \left( \frac{\pi}{4}, 1 \right) \), \( \left( \frac{\pi}{2}, a \right) \)

**“a” represents the vertical asymptote**

Period: \( \pi \)

Domain: \( \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \)

Range: \( (-\infty, \infty) \)

The **DOMAIN** uses ( ) because of the vertical asymptotes.

The **RANGE** uses ( ) because of the lack of amplitude (no minimum no maximum)

TRANSFORMATIONS OF THE PARENT FUNCTION OF TANGENT

**EXAMPLE 1:** \( y = \tan 2x - 3 \)

Transformation(s): Horoizontal Dilation compressed by a factor of \( \frac{1}{2} \), Vertical Shift down 2

Period: \( \frac{\pi}{2} \)

Amplitude: none

Domain: \( \left( -\frac{\pi}{4}, \frac{\pi}{4} \right) \)

Range: \( (-\infty, \infty) \)

Significant points:
- \( \left( -\frac{\pi}{4}, a \right) \), \( \left( -\frac{\pi}{8}, -4 \right) \), \( (0, -3) \), \( \left( \frac{\pi}{8}, -2 \right) \), \( \left( \frac{\pi}{4}, a \right) \)

\[\begin{array}{cccccc}
(-\pi/2, a) & (-\pi/4, -1) & (0, 0) & (\pi/4, 1) & (\pi/2, a) \\
\star \frac{1}{2} & \star \frac{1}{2} & \star \frac{1}{2} & \star \frac{1}{2} & \star \frac{1}{2} \\
(-\pi/4, a) & (-\pi/8, -1) & (0, 0) & (\pi/8, 1) & (\pi/4, a) \\
-3 & -3 & -3 & -3 & -3 \\
(-\pi/4, a) & (-\pi/8, -4) & (0, -3) & (\pi/8, -2) & (\pi/4, a)
\end{array}\]
**EXAMPLE 2:** \( y = 2 \tan(x - \pi) \)

**Transformation(s):** Vertical Dilation stretched by a factor of 2, Horizontal Shift right \( \pi \)

- **Period:** \( \frac{\pi}{2} \)  
- **Amplitude:** none  
- **Domain:** \( \left( \frac{\pi}{2}, \frac{3\pi}{2} \right) \)  
- **Range:** \( (-\infty, \infty) \)

**Period:** \( \frac{\pi}{b} \), \( b = 1 \) therefore \( \frac{\pi}{1} = \pi \)

**Significant points:** \( \left( \frac{\pi}{2}, a \right), \left( \frac{3\pi}{4}, -2 \right), (\pi, 0), \left( \frac{5\pi}{4}, 2 \right), \left( \frac{3\pi}{2}, a \right) \)

\[
\begin{array}{cccccccc}
(-\frac{\pi}{2}, a) & (-\frac{\pi}{4}, -1) & (0, 0) & (\frac{\pi}{4}, 1) & (\frac{\pi}{2}, a) \\
\times 2 & \times 2 & \times 2 & \times 2 & \times 2 \\
(-\frac{\pi}{2}, a) & (-\frac{\pi}{4}, -2) & (0, 0) & (\frac{\pi}{4}, 2) & (\frac{\pi}{2}, a) \\
+\pi & +\pi & +\pi & +\pi & +\pi \\
(\frac{\pi}{2}, a) & (\frac{3\pi}{4}, -2) & (\pi, 0) & (\frac{5\pi}{4}, 2) & (\frac{3\pi}{2}, a)
\end{array}
\]
COSINE & TANGENT FUNCTIONS ASSIGNMENT

Given the function, identify the transformation(s), period, and amplitude. Clearly show work to find the significant points. Sketch the graph over one complete period on a clearly labeled grid and identify the domain and range.

1. \( g(x) = \cos \frac{1}{3} (x + \frac{\pi}{2}) \)
   Transformation(s): ______________________________
   _____________________________________________
   Period: _______  Amplitude: _______
   Domain: _______  Range: _______
   Significant points:
   _____________________________________________

2. \( f(x) = 3 \sin \frac{1}{2} x - 2 \)
   Transformation(s): ______________________________
   _____________________________________________
   Period: _______  Amplitude: _______
   Domain: _______  Range: _______
   Significant points:
   _____________________________________________

Sketch the angle and then evaluate the trig function. EXACT answers ONLY. Show work where necessary.

3. \( \cos \frac{15\pi}{4} \)  
   4. \( \tan 180^\circ \)  
   5. \( \sin \frac{2\pi}{3} \)
Given the function, identify the transformation(s), period, and amplitude. Clearly show work to find the significant points. Sketch the graph over one complete period on a clearly labeled grid and identify the domain and range.

6. \( y = \tan(x + \frac{\pi}{2}) \)

Transformation(s): ______________________________
_________________________________________________
Period: _______   Domain: _______   Range: _______

Significant points:
_________________________________________________

7. \( y = 2 \cos x + 1 \)

Transformation(s): ______________________________
_________________________________________________
Period: _______   Amplitude: _______
Domain: _______   Range: _______
Significant points:
_________________________________________________

Write the sine equation for each graph.

8. \[ \text{EQUATION: } \]

\( a = _____ \quad b = _____ \)

\( c = _____ \quad d = _____ \)
Write the sine equation for each graph.

9. \[ \text{EQUATION: } \frac{\sin(x - c)}{d} = a \]
   \[ a = \_ \quad b = \_ \]
   \[ c = \_ \quad d = \_ \]

10. \[ \text{EQUATION: } \frac{\sin(x - c)}{d} = a \]
    \[ a = \_ \quad b = \_ \]
    \[ c = \_ \quad d = \_ \]
SOLVING TRIGONOMETRIC EQUATIONS

To solve a trigonometric equation, use standard algebraic techniques such as:

1. Collecting and combining like terms
2. Using inverse operations (the process of do/undo for solving an equation)
3. Factoring

The goal to solving a trigonometric equation is to isolate the trigonometric function in the equation.

Solving trigonometric equations in the interval \([0, 2\pi]\) involves one (1) rotation around the unit circle.

In order to solve trigonometric equations successfully, you will need to have and use your unit circle.

The answers should be exact, so avoid decimal solutions.

SOLVING BASIC TRIG EQUATIONS

EXAMPLE 1: Solve the trigonometric equation in the interval \([0, 2\pi]\).

Steps:

\[
2 \sin x - \sqrt{3} = 0
\]

Use inverse operations to move \(\sqrt{3}\)

\[
+\sqrt{3} + \sqrt{3}
\]

Add \(\sqrt{3}\) to both sides of the equation

\[
2 \sin x = \sqrt{3}
\]

Use inverse operations to move 2

\[
2 \quad 2
\]

Divide by 2 on both sides of the equation

\[
\sin x = \frac{\sqrt{3}}{2}
\]

THIS IS NOT THE SOLUTION!

Remember that sine is the y-value of the coordinate on the unit circle.

Solving for \(x\) means using the Unit Circle to find the angle(s) where the y-value is \(\frac{\sqrt{3}}{2}\)

The angles where the y-value is \(\frac{\sqrt{3}}{2}\) are \(\frac{\pi}{3}\) and \(\frac{2\pi}{3}\), therefore, \(x = \frac{\pi}{3}, \frac{2\pi}{3}\)
EXAMPLE 2: Solve the trigonometric equation in the interval \([0, 2\pi)\).

\[
2 + 3 \cos x - 5 = 0
\]

Steps:

\[
2 + 3 \cos x - 5 = 0 \quad \text{Combine like terms}
\]

\[
-3 \cos x - 3 = 0 \quad \text{Use inverse operations to move } -3
\]

\[
+3 + 3
\]

\[
-3 \cos x = 3
\]

\[
-3 \cos x = 3
\]

\[
\cos x = -1
\]

Remember that cosine represents the \(x\)-value of the coordinate on the unit circle. There is only one angle where \(\cos x = -1\), which is \(\pi\). Therefore, the solution is \(x = \pi\).

EXAMPLE 3: Solve the trigonometric equation in the interval \([0, 2\pi)\).

\[
5 \tan^3 x - 5 = 0
\]

Steps:

\[
5 \tan^3 x - 5 = 0 \quad \text{Use inverse operations to move } -5
\]

\[
+5 + 5
\]

\[
5 \tan^3 x = 5
\]

\[
5 \tan^3 x = 5
\]

\[
\tan^3 x = 1
\]

\[
\tan^3 x = \sqrt[3]{1}
\]

\[
\tan x = 1
\]

There are 2 angles where the tangent equals 1, which are \(\frac{\pi}{4}\) and \(\frac{5\pi}{4}\). Therefore, \(x = \frac{\pi}{4}, \frac{5\pi}{4}\).
YOU TRY

1. \( \sin x + \sqrt{2} = -\sin x \)

2. \( 4 \cos^2 x - 3 = 0 \)

3. \( 3 \tan^2 x - 9 = 0 \)

SOLVING TRIG EQUATIONS BY FACTORING

- When an equation is in the format \( au^2 + bu + c \) where \( u \) is the trig function of \( x \), you can **factor** to solve the equation. Write the trig equation like a quadratic equation by removing the trig functions (sin, cos, tan)

**EXAMPLE 1:** Solve the trig equation \( 2 \sin^2 x + 3 \sin x + 1 = 0 \) in the interval \([0, 2\pi)\).

Original Trig Equation

```
2 \sin^2 x + 3 \sin x + 1 = 0
```

Parallel Quadratic Equation

```
2x^2 + 3x + 1 = 0
```

Factor

```
x(2x + 1) + 1(2x + 1) = 0
```

```
(x + 1)(2x + 1) = 0
```

Set each factor equal to 0 and solve for \( x \).

```
\sin x + 1 = 0          2 \sin x + 1 = 0
\hline
-1 | -1                     -1 | -1
\sin x = -1             2 \sin x = -1
\hline
2 | 2                         2 | 2
\sin x = -\frac{1}{2}
```

The solution(s) are the angle(s) where \( \sin \) (the \( y \)-value) on the unit circle equals \(-1\) and \(-\frac{1}{2}\). Therefore, \( x = \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \).
EXAMPLE 2: Solve the trig equation \(2 \cos^2 x + \cos x = 1\) in the interval \([0, 2\pi)\).

\[2 \cos^2 x + \cos x = 1\]  \hspace{1cm} \text{Parallel Equation} \quad \rightarrow \quad 2x^2 + x = 1 \quad \text{Put in standard form}

\[(2 \cos x - 1)(\cos x + 1) = 0\]  \hspace{1cm} \text{Set equal to 0 and solve}

\[2x^2 + x - 1 = 0\]  \hspace{1cm} \text{Factor}

\[(2x - 1)(x + 1) = 0\]  \hspace{1cm} \text{Use for trig equation}

\[2 \cos^2 x - 1 = 0\]  \hspace{1cm} \cos x + 1 = 0

\[\cos x = \frac{1}{2}\]  \hspace{1cm} \cos x = -1

The angles on the unit circle where the \(x\)-values are equal to \(\frac{1}{2}\) and \(-1\) are \(\frac{\pi}{3}\), \(\frac{5\pi}{3}\), and \(\pi\). Therefore, the solutions are \(x = \frac{\pi}{3}\); \(\frac{5\pi}{3}\); \(\pi\).

Since the equation uses cosine, find the angle(s) on the unit circle where the \(x\)-value equals \(\frac{1}{2}\) & \(-1\).
SOLVING TRIGONOMETRIC EQUATIONS (PART 1) ASSIGNMENT

Solve over \([0, 2\pi]\). Must show work for credit.

1. \( \sin x + \sqrt{2} = -\sin x \)

2. \( 3 \tan^2 x - 1 = 0 \)

3. \( 2 \cos x - \sqrt{3} = 0 \)

4. \( 2 \cos^2 x - 3 \cos x + 1 = 0 \)

5. \( 4 \sin^2 x - 3 = 0 \)

6. \( \tan^2 x = 1 \)
7. \(2 \cos^2 x - \cos x = 1\)  
8. \(5 + 2 \sin x - 7 = 0\)

9. \(3 \cos x = \cos x - 1\)  
10. \(2 \sin^2 x - 1 = 0\)

11. \(4 \cos^2 x - 1 = 0\)  
12. \(5 \tan^2 x - 15 = 0\)
SOLVING TRIG EQUATIONS WITH MORE THAN ONE TRIG FUNCTION

- To solve trig equations that have more than one trig function, you need a **Trigonometric Identity or Formula**

  - There are two types of Trigonometric Identities/Formulas we will use:
    - **Pythagorean Identities**
    - **Double Angle Identities**

**PART 1:**

There are many Pythagorean Identities, however, we will focus on only one identity and its transformations:

\[
\sin^2 x + \cos^2 x = 1
\]

**Pythagorean Identity**

**Transformation of Identity**

\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1 \\
- \cos^2 x - \cos^2 x &= 1 - \cos^2 x \\
\sin^2 x &= 1 - \cos^2 x
\end{align*}
\]

**Transformation of Identity**

\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1 \\
- \sin^2 x - \sin^2 x &= 1 - \sin^2 x \\
\cos^2 x &= 1 - \sin^2 x
\end{align*}
\]

Sometimes it is possible to use the Pythagorean Identity as it is and sometimes you must use one of the transformations.
EXAMPLE 1: Solve the trig equation $2 \cos^2 x + 3 \sin x - 3 = 0$ in the interval $[0, 2\pi)$.

1. All of the trig functions need to be the same.

2. Since there is an identity that contains $\cos^2 x$ and not $\sin x$, substitute $\cos^2 x$ with $1 - \sin^2 x$ so that the equation only contains sine.

3. Distribute the 2 and put the equation in standard form.

4. Write a parallel problem and factor it.

5. Set both factors equal to 0 and solve.

6. Using the unit circle, find the angles where sine (the y-value) is $\frac{1}{2}$ and 1.

The angles where sine is $\frac{1}{2}$ and 1 are $\frac{\pi}{6}$, $\frac{5\pi}{6}$, and $\frac{\pi}{2}$.

Therefore, $x = \frac{\pi}{6}$, $\frac{5\pi}{6}$, and $\frac{\pi}{2}$. 

EXAMPLE 2: Solve the trig equation \( \cos^2 x - \cos x + 1 = \sin^2 x \) in the interval \([0, 2\pi)\).

\[
\begin{align*}
\cos^2 x - \cos x + 1 &= \sin^2 x \\
\cos^2 x - \cos x + 1 &= (1 - \cos^2 x) \\
\cos^2 x - \cos x + 1 &= 1 - \cos^2 x \\
-1 - 1 &= - \cos^2 x \\
\cos^2 x - \cos x &= - \cos^2 x \\
+ \cos^2 x + \cos^2 x &= 2 \cos^2 x - \cos x = 0 \\
\cos x (2 \cos x - 1) &= 0 \\
\cos x &= 0 \\
2 \cos x - 1 &= 0 \\
+ 1 + 1 &= 2 \cos x = 1 \\
2 &= 2 \\
\cos x &= \frac{1}{2}
\end{align*}
\]

The equation has a \( \cos x \) in it & we don’t have an identity with \( \cos x \), so we must substitute the \( \sin^2 x \) using the transformation \( \sin^2 x = 1 - \cos^2 x \).

Write a parallel problem (quadratic equation) and factor.

\[
2x^2 - x = 0 \\
x (2x - 1) = 0
\]

On the unit circle, the angles where cosine (the \( x \)-value) equals 0 and \( \frac{1}{2} \) are \( \frac{\pi}{3} \), \( \frac{5\pi}{3} \), \( \frac{\pi}{2} \) and \( \frac{3\pi}{2} \).

Therefore, \( x = \frac{\pi}{3} ; \frac{5\pi}{3} ; \frac{\pi}{2} \), \( \frac{3\pi}{2} \).

PART 2:

Another type of Identity/Formula used to solve Trig Equations are Double Angle Formulas.

Double Angle Formulas are used to solve Trig Equations where the measure of the angle is \( 2a \).

<table>
<thead>
<tr>
<th>Formula</th>
<th>( \sin 2x = 2 \sin x \cos x )</th>
<th>( \cos 2x = \cos^2 x - \sin^2 x )</th>
<th>( \tan 2x = \frac{2 \tan x}{1-\tan^2 x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformation of Original Formula</td>
<td>None</td>
<td>( \cos 2x = 2 \cos^2 x - 1 )</td>
<td>( \tan 2x = \frac{\sin 2x}{\cos 2x} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \cos 2x = 1 - 2 \sin^2 x )</td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE 1: Solve the trig equation \( \sin 2x + 2 \cos x = 0 \) over the interval \([0, 2\pi)\).

\[ \sin 2x + 2 \cos x = 0 \]  
1. Substitute for the Double Angle, \( \sin 2x \).

\[ \sin 2x = 2 \sin x \cos x \]  
There is only 1 formula: \( \sin 2x = 2 \sin x \cos x \)

\[ (2 \sin x \cos x) + 2 \cos x = 0 \]  
2. Although there are different trig functions in the equation, it can be solved. Let \( \sin x \) be represented by \( x \) and \( \cos x \) be represented by \( y \). Write a parallel problem and factor.

\[ 2x y + 2y = 0 \ (\text{Factor out GCF}) \]  
\[ 2y(x + 1) = 0 \]

Substitute back into the trig equation.

\[ \frac{2 \cos x}{2} = 0 \quad \frac{\sin x + 1}{1} = 0 \]  
3. Set each factor equal to 0 and solve.

\[ \cos x = 0 \quad \sin x = -1 \]

On the unit circle, find the angles where cosine (\( x \)-value) equals 0 and sine (\( y \)-value) equals -1.

The angles where \( \cos x = 0 \) and \( \sin x = -1 \) are \( \frac{\pi}{2} \) and \( \frac{3\pi}{2} \).

Therefore, \( x = \frac{\pi}{2} ; \frac{3\pi}{2} \).
EXAMPLE 2: Solve the trig equation \( \tan 2x + \tan x = 0 \) over the interval \([0, 2\pi]\).

\[
\begin{align*}
\frac{\tan 2x}{1} + \tan x &= 0 \\
\frac{2\tan x}{1-\tan^2 x} + \tan x &= 0 \\
-\tan x - \tan x &= 0 \\
\frac{2\tan x}{1-\tan^2 x} &= -\tan x \\
(1 - \tan^2 x) \cdot \frac{2\tan x}{1-\tan^2 x} &= -\tan x \cdot (1 - \tan^2 x) \\
2\tan x &= -\tan x (1 - \tan^2 x) \\
2\tan x &= -\tan x + \tan^3 x \\
-2\tan x &= \tan x \\
0 &= \tan^3 x - 3\tan x \\
0 &= \tan x (\tan^2 x - 3) \\
\tan x &= 0 \\
\tan^2 x - 3 &= 0 \\
0 &= x^3 - 3x \\
0 &= x(x^2 - 3) \\
0 &= x^3 - 3x \\
0 &= x(x^2 - 3) \\
\end{align*}
\]

Move \( \tan x \) to the right side of the equation and eliminate the fraction on the left by multiplying both sides by the denominator.

Distribute \( \tan x \) and move all terms to the same side of the equation. Then write a parallel problem and factor.

On the unit circle, locate the angles where tangent equals \( 0, \sqrt{3}, \text{ and } -\sqrt{3} \).

The angles where tangent equals \( 0, \sqrt{3}, \text{ and } -\sqrt{3} \) are \( 0\pi, \frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \text{ and } \frac{5\pi}{3} \).

Therefore, \( x = 0\pi; \pi; \frac{\pi}{3}; \frac{4\pi}{3}; \frac{2\pi}{3}; \frac{5\pi}{3} \).
SOLVING TRIGONOMETRIC EQUATIONS (PART 2) ASSIGNMENT

Solve over [0, 2π). Must show work for credit.

1. \( \sin^2 x + 2 \sin x + 1 = \cos^2 x \)  
2. \( \sin^2 x = 3 \cos^2 x \)

3. \( \cos^2 x + 2 \cos x + 1 = \sin^2 x \)  
4. \( 3 \tan^3 x - 3 \tan^2 x - \tan x + 1 = 0 \)

5. \( \sin 2x - \cos x = 0 \)  
6. \( 2 \cos^2 x + 3 = 7 \cos x \)
7. $\cos 2x + \cos x = 0$

8. $\sqrt{2}\cos x \sin x - \cos x = 0$

9. $5\cos x - \sqrt{3} = 3\cos x$

10. $\cos 2x + \sin x = 0$

11. $3\tan^2 x = 1$

12. $2\sin^2 x = 2 + \cos x$