IB Math Studies – Unit 8 Test (for week of 3/31/2020 – 4/6/2020)

1. The function \( f(x) \) is defined by \( f(x) = 1.5x + 4 + \frac{6}{x}, x \neq 0 \).

   (a) Write down the equation of the vertical asymptote.  

   (2)

   (b) Find \( f'(x) \).  

   (3)

   (c) Find the gradient of the graph of the function at \( x = -1 \).  

   (2)

   (d) Using your answer to part (c), decide whether the function \( f(x) \) is increasing or decreasing at \( x = -1 \). Justify your answer.  

   (2)

   (e) Sketch the graph of \( f(x) \) for \(-10 \leq x \leq 10 \) and \(-20 \leq y \leq 20 \).  

   (4)

   \( P_1 \) is the local maximum point and \( P_2 \) is the local minimum point on the graph of \( f(x) \).

   (f) Using your graphic display calculator, write down the coordinates of

       (i) \( P_1 \);

       (ii) \( P_2 \).  

   (4)

   (g) Using your sketch from (e), determine the range of the function \( f(x) \) for \(-10 \leq x \leq 10 \).  

   (3)

(Total 20 marks)
2. The table given below describes the behavior of \( f'(x) \), the derivative function of \( f(x) \), in the domain \(-4 < x < 2\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4 &lt; x &lt; -2)</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>(-2)</td>
<td>0</td>
</tr>
<tr>
<td>(-2 &lt; x &lt; 1)</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>(1 &lt; x &lt; 2)</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

(a) State whether \( f(0) \) is greater than, less than or equal to \( f(-2) \). Give a reason for your answer.

(b) The point \( P(-2, 3) \) lies on the graph of \( f(x) \).

(c) From the information given about \( f'(x) \), state whether the point \((-2, 3)\) is a maximum, a minimum or neither. Give a reason for your answer.
The diagram shows a sketch of the function \( f(x) = 4x^3 - 9x^2 - 12x + 3 \).

(a) Write down the values of \( x \) where the graph of \( f(x) \) intersects the \( x \)-axis.

(b) Write down \( f'(x) \).

(c) Find the value of the local maximum of \( y = f(x) \).

Let \( P \) be the point where the graph of \( f(x) \) intersects the \( y \)-axis.

(d) Write down the coordinates of \( P \).

(e) Find the gradient of the curve at \( P \).

The line, \( L \), is the tangent to the graph of \( f(x) \) at \( P \).

(f) Find the equation of \( L \) in the form \( y = mx + c \).

There is a second point, \( Q \), on the curve at which the tangent to \( f(x) \) is parallel to \( L \).

(g) Write down the gradient of the tangent at \( Q \).

(h) Calculate the \( x \)-coordinate of \( Q \).

(Total 19 marks)
4. A dog food manufacturer has to cut production costs. She wishes to use as little aluminum as possible in the construction of cylindrical cans. In the following diagram, \( h \) represents the height of the can in cm, and \( x \) represents the radius of the base of the can in cm.

The volume of the dog food cans is 600 cm\(^3\).

(a) Show that \( h = \frac{600}{\pi x^2} \).

(b) (i) Find an expression for the curved surface area of the can, in terms of \( x \). Simplify your answer.

(ii) Hence write down an expression for \( A \), the total surface area of the can, in terms of \( x \).

(c) Differentiate \( A \) in terms of \( x \).

(d) Find the value of \( x \) that makes \( A \) a minimum.

(e) Calculate the minimum total surface area of the dog food can.

(Total 14 marks)
1. Given \( f(x) = \frac{5}{x^3} - \frac{2}{x^2} + \frac{3}{x} - 6x + 1 \),
   
   (a) Calculate \( f'(x) \).
   
   (b) Find \( f'(-1) \).
   
   (c) Explain what the value of \( f'(-1) \) represents.

2. Let \( p \) stand for the proposition “I will wear a hat.” Let \( q \) stand for the proposition “it is cloudy.”
   
   (a) Write the following statements in symbolic logic form:
       
       (i) “I will wear a hat is and only if it is not cloudy.”

       (ii) “Either I will not wear a hat or it will be cloudy, but not both.”

   (b) Write down, in words, the contrapositive of the statement, “If it is cloudy, then I will wear a hat.”

3. The table below shows the number of men and women in a small town who voted in a local election.

<table>
<thead>
<tr>
<th></th>
<th>Voted</th>
<th>Did not vote</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>57</td>
<td>10</td>
<td>67</td>
</tr>
<tr>
<td>Female</td>
<td>45</td>
<td>18</td>
<td>63</td>
</tr>
<tr>
<td>Total</td>
<td>102</td>
<td>28</td>
<td>130</td>
</tr>
</tbody>
</table>

   (a) If a person was selected at random from the town, find the probability that the person

   (i) voted and is female,

   (ii) did not vote,

   (iii) did not vote, given that the person selected is male.

   (b) If two randomly selected people were selected from the town, find the probability that both voted.
4. A bag contains 6 red and 4 green candies.

Pauline randomly selects one sweet out of the bag and eats it. Then she randomly selects a second sweet. Below is a tree diagram showing Paulina’s possible choices. Two of the probability values are missing.
(a) Fill in the missing probability values on the tree diagram.

(b) If Paulina eats two candies, what is the probability that she will eat at least one green?
(c) What is the probability Paulina will select a red candy given the first was green?

5. Paige is training for an endurance cycling challenge. She rides 1.5 kilometers in her first week of training, 2.25 kilometers during the second week, 3 kilometers during the third, and so on.
(a) Calculate the number of kilometers Paige rides during her tenth week of training.
(b) Find the total number of kilometers ridden by the end of her tenth week of training.

The cycling challenge involves riding 125 kilometers in one day.
(c) Determine the number of training weeks Paige must ride before she has ridden a total of 125 kilometers.

6. A professor surveyed 200 recent college graduates to determine if the degree obtained was independent of employment status. The majors were engineering, education, marketing, accounting, and computer science. The graduates were either employed or unemployed. A $\chi^2$ test was conducted at the 5% significance level.
(a) Write down the null hypothesis.
(b) Find the number of degrees of freedom for this test.
(c) If the calculated $p$-value was 0.032, determine if the degree obtained is independent of employment status.
   Give a clear reason for your answer.
7. The data below display the number of cars parked at a shopping center on 21 randomly selected days last month.

<table>
<thead>
<tr>
<th>5</th>
<th>8</th>
<th>13</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>28</td>
<td>31</td>
<td>33</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>40</td>
<td>41</td>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

(a) Determine the
   (i) lower quartile,
   (ii) median,
   (iii) upper quartile.

(b) Complete the frequency table below.

<table>
<thead>
<tr>
<th>Number of Cars</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 10</td>
<td></td>
</tr>
<tr>
<td>11 – 20</td>
<td></td>
</tr>
<tr>
<td>21 – 30</td>
<td></td>
</tr>
<tr>
<td>31 – 40</td>
<td></td>
</tr>
<tr>
<td>41 – 50</td>
<td></td>
</tr>
</tbody>
</table>

(c) State whether the data is discrete or continuous.

8. Let $X$ be normally distributed with a mean of 75 and a standard deviation of 10.

(a) On the diagram below, shade the area representing $P(X < 65)$.

(b) Calculate the area of the shaded region above.

(c) Find $P(65 < X < 95)$. 
9. A boat is 450 meters from the base of a cliff. The angle of elevation from the boat to the top of the cliff is 25°.

(a) Draw a diagram representing the situation. Clearly label the distance and the angle given.

(b) Find the height of the cliff in kilometers.

The boat moves closer to the base of the cliff such that the angle of elevation increases to 30°.

(c) Determine the distance traveled by the boat.

10. Ariana was in charge of a game for children at a school festival. Children tossed a bean bag from a set distance and tried to land the bean bag in a small basket. They could toss the bean bag as many times as they needed in order to win a piece of candy.

During a one-hour period, she recorded the number of tosses it took each child before winning.

The results are shown in the frequency table below.

<table>
<thead>
<tr>
<th>Number of Tosses</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 3</td>
<td>3</td>
</tr>
<tr>
<td>4 – 6</td>
<td>6</td>
</tr>
<tr>
<td>7 – 9</td>
<td>9</td>
</tr>
<tr>
<td>10 – 12</td>
<td>5</td>
</tr>
<tr>
<td>13 – 15</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Calculate an approximate

(i) mean,

(ii) standard deviation.

(b) What is the probability a child would need more than nine tosses to win?

(c) Suppose the number of tosses follows a normal distribution. What is the probability a randomly selected child would need fewer than five tosses?