Let \( f(x) = \frac{1}{\sqrt{2x-1}} \), for \( x > \frac{1}{2} \).

(a) Find \( \int (f(x))^2 \, dx \).

(b) Part of the graph of \( f \) is shown in the following diagram.

The shaded region \( R \) is enclosed by the graph of \( f \), the \( x \)-axis, and the lines \( x = 1 \) and \( x = 9 \). Find the volume of the solid formed when \( R \) is revolved 360° about the \( x \)-axis.
1.

(a) Find \( \int \frac{e^x}{1 + e^x} \, dx \).

(b) Find \( \int \sin 3x \cos 3x \, dx \).

2.

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building. The angle of depression from T to a point A on the horizontal ground is 35°. The angle of elevation of the top of the building from A is 30°.  

Find the height of the building.
1.

A circle centre O and radius $r$ is shown below. The chord $[AB]$ divides the area of the circle into two parts. Angle $AOB$ is $\theta$.

(a) Find an expression for the area of the shaded region.

(b) The chord $[AB]$ divides the area of the circle in the ratio $1:7$. Find the value of $\theta$.

2.

Let $f(x) = ax^2 - 4x - c$. A horizontal line, $L$, intersects the graph of $f$ at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find $p$.

(ii) Hence, show that $a = 2$.

(b) The equation of $L$ is $y = 5$. Find the value of $c$. 
1. A toy car travels with velocity $v \text{ ms}^{-1}$ for six seconds. This is shown in the graph below.

![Graph of velocity v vs time t](image)

a) Write down the car's velocity at $t = 3$. 
B) Find the car's acceleration at $t = 1.5$

c) Find the total distance travelled.

2. The expression $6 \sin x \cos x$ can be expressed in the form $a \sin bx$.

a) Find the value of $a$ and of $b$.

b) Hence or otherwise, solve the equation $6 \sin x \cos x = \frac{3}{2}$, for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

3. Given that $f(x) = \frac{1}{x}$, answer the following

a) Find the first four derivatives of $f(x)$.

b) Write an expression for $f^n(x)$ in terms of $x$ and $n$. 
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The first 3 terms in a geometric sequence are: $k^2$, $-k$, $k - 2$.

a) Find the value of $k$.

b) Find the sum of the terms to infinity.

Find the $x^3$ term in the expansion of $(x - \frac{2}{x})^5$.

2.  [Maximum mark: 6]

The following Venn diagram shows the events $A$ and $B$, where $P(A) = 0.4$, $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.1$. The values $p$ and $q$ are probabilities.

(a) (i) Write down the value of $q$.

(ii) Find the value of $p$.

(b) Find $P(B)$.
The graph of the function \( f(x) = a \sin (bx) + c \) is shown below for \(-360^\circ \leq x \leq 1080^\circ\).

(a) Write down the period of \( f(x) \).

(b) Write down the value of

(i) \( a \);
(ii) \( b \);
(iii) \( c \).

\( P \) is one of the points where the graph \( y = f(x) \) intersects the \( x \)-axis. The \( x \)-coordinate of \( P \) lies between \(-180^\circ \) and \( 180^\circ \).

(c) (i) Mark and label the point \( P \) on the graph above.
(ii) \textbf{Estimate} the \( x \)-coordinate of \( P \).
8. [Maximum mark: 16]

Let \( \mathbf{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} \) and \( \mathbf{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \).

(a) (i) Find \( \mathbf{AB} \).

(ii) Find \( |\mathbf{AB}| \). \[4\]

The point C is such that \( \mathbf{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \).

(b) Show that the coordinates of C are \((-2, 1, 3)\). \[1\]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle \( \angle ADC = \theta \).

![Diagram of triangle ABC with point D on line segment BC]

(c) Write down an expression in terms of \( \theta \) for

(i) angle ADB;

(ii) area of triangle ABD. \[2\]

(d) Given that \( \frac{\text{area } \triangle ABD}{\text{area } \triangle ACD} = 3 \), show that \( \frac{BD}{BC} = \frac{3}{4} \). \[5\]

(e) Hence or otherwise, find the coordinates of point D. \[4\]
10. [Maximum mark: 16]

Let \( f(x) = \cos x \).

(a) (i) Find the first four derivatives of \( f(x) \).

(ii) Find \( f^{(19)}(x) \). \[4\]

Let \( g(x) = x^k \), where \( k \in \mathbb{Z}^+ \).

(b) (i) Find the first three derivatives of \( g(x) \).

(ii) Given that \( g^{(19)}(x) = \frac{k!}{(k-p)!}(x^{k-19}) \), find \( p \). \[5\]

Let \( k = 21 \) and \( h(x) = \left( f^{(19)}(x) \times g^{(19)}(x) \right) \).

(c) (i) Find \( h'(x) \).

(ii) Hence, show that \( h'(\pi) = \frac{-21!}{2} \pi^2 \). \[7\]
Point A has coordinates \((-4, -12, 1)\) and point B has coordinates \((2, -4, -4)\).

(a) Show that \(\vec{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix} \).

(b) The line \(L\) passes through A and B.

(i) Find a vector equation for \(L\).

(ii) Point \(C(k, 12, -k)\) is on \(L\). Show that \(k = 14\).

(c) (i) Find \(\vec{OB} \cdot \vec{AB}\)

(ii) Write down the value of angle \(OBA\).

Point D is also on \(L\) and has coordinates \((8, 4, -9)\).

(d) Find the area of triangle \(OCD\).

7. [Maximum mark: 7]

Consider \(f(x), g(x)\) and \(h(x)\), for \(x \in \mathbb{R}\) where \(h(x) = (f \circ g)(x)\).

Given that \(g(3) = 7, g'(3) = 4\) and \(f''(7) = -5\), find the gradient of the normal to the curve of \(h\) at \(x = 3\).
1.  [Maximum mark: 7]

In the following diagram, $u = \overrightarrow{AB}$ and $v = \overrightarrow{BD}$.

[Diagram showing triangle ABC with points A, B, C, D, E, and vectors u and v defined as follows: u from A to B, v from B to D, E is the midpoint of AD, BD/DC = 1/3.]

The midpoint of $\overrightarrow{AD}$ is E and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of $u$ and $v$.

(a) $\overrightarrow{AE}$

(b) $\overrightarrow{EC}$

2.

Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leq 5$.

Points $P(0.25, 0)$ and $Q$ are on the curve of $f$. The tangent to the curve of $f$ at $P$ is perpendicular to the tangent at $Q$. Find the coordinates of $Q$. 