Anticipatory Set - Solve the following linear equations

1) \(3x - 9 = 3\)
\[\begin{align*}
-9 + 9 &= 0 \\
3x &= 12 \\
\frac{3x}{3} &= \frac{12}{3} \\
x &= 4
\end{align*}\]

2) \(\frac{1}{2}x + 4 = 6\)
\[\begin{align*}
-4 - 4 &= 0 \\
\frac{1}{2}x &= 2 \\
\frac{2}{1}x &= \frac{2}{1} \cdot 2 \\
x &= 4
\end{align*}\]

3) \(\frac{2}{3}x - \frac{1}{3} = 3\)
\[\begin{align*}
\frac{2}{3}x &= \frac{3}{1} + \frac{1}{3} \\
\frac{2}{3}x &= \frac{10}{3} \\
\frac{3}{2} \cdot \frac{2}{3}x &= \frac{3}{2} \cdot \frac{10}{3} \\
x &= \frac{15}{3} \\
x &= 5
\end{align*}\]

4) \(\frac{1}{3}x - 2 = 10\)
\[\begin{align*}
\frac{1}{3}x &= 2 + 10 \\
\frac{3}{1} \cdot \frac{1}{3}x &= \frac{3}{1} \cdot 2 + 10 \\
x &= 3 \\
x &= \frac{15}{3} \\
x &= 5
\end{align*}\]

What is the absolute value of a number?
The absolute value of a number is that number's distance from zero on the number line.

Example: \(|4| = 4\)
Example: \(|-3| = 3\)

Notice: We can take the absolute value of positive numbers, negative numbers and zero, but the absolute value of a number cannot be negative.

The absolute value symbol "| |" is a grouping symbol similar to parentheses "()"
The absolute value is in the same place in the order of operations (PEMDAS) as parentheses.

P- Parentheses and absolute value
E- Exponents
MD- Mult/Div
AS- Add/Sub

Solving Absolute Value Equations
What value(s) of \(x\) will make the following equation true?
\(|x| = 4\)
\[\begin{align*}
|4| &= 4 \\
x &= 4 \\
x &= -4
\end{align*}\]

Solve the following equation for \(x\)
\(|2x + 8| + 2 = 14\)
\[\begin{align*}
|2x+8| &= 12 \\
2x + 8 &= 12 \\
2x &= 4 \\
x &= 2
\end{align*}\]

Graphically:
Verbally: \(x = -10\) or \(x = 2\)
Set Builder Notation: \(\{x | x = -2, \text{or} \ x = 10\}\)

Solving absolute value equations procedures
Step 1) Isolate the absolute value
\[3|x - 4| = 6\]
\[|x - 4| = 2\]
Step 2) Write two equations
\[x - 4 = 2 \quad \text{or} \quad x - 4 = -2\]
Step 3) Solve both equations
\[x - 4 = 2 \\
x = 6\]
\[x - 4 = -2 \\
x = 2\]

Step 4) Check both solutions in the original equation
\[3(6) - 4| = ? \quad \text{or} \quad 3(2) - 4| = ?\]
\[\text{Check} \quad \text{Graphically:} \]
Verbally: \(x = 7\) or \(x = 1\)
Set Builder Notation: \(\{x | x = 7, \text{or} \ x = 1\}\)

Solve for \(x\):
\[3|2x - 8| + 9 = 21\]
\[2|2x - 8| = 12\]
\[|2x - 8| = 6\]
\[2x - 8 = \pm 6\]
\[2x = 14 \quad \text{or} \quad 2x = 2\]
\[x = 7 \quad \text{or} \quad x = 1\]

Graphically:
Verbally: \(x = 7\) or \(x = 1\)
Set Builder Notation: \(\{x | x = 7, \text{or} \ x = 1\}\)
Solve the equation

\[ |x - 3| + 1 = 2x + 1 \]

\[ \frac{-1}{\overline{1x - 3| = 2x}} \]

\[ x - 3 = 2x \]
\[ -3 = x \]

\[ \text{extraneous} \]

**Special Case**

If you are left with a negative number on one side of the equation after you isolate the absolute value, then the **equation has no solutions!!!!**

Example \(|3x + 1| = -3\)

\[ \text{will never be negative} \]

What is the solution to the following equation

\[ |x| = -4 \]

\[ \text{will never be} \]

\[ -4 \]

Graphically:

![Graphical representation]

Verbally: \( x = 1 \)

Set Builder Notation: \( \{ x | x = 1 \} \)

**POU:** Give an explanation of why both of the following equations have been solved incorrectly:

1. \( |x + 4| = 3 \)
   - \( x + 4 = 3 \) or \( x + 4 = -3 \)
   - \( x = -1 \) or \( x = -7 \)

2. \( |x + 7| = 5 \)
   - \( x + 7 = 5 \) or \( x - 7 = -5 \)
   - \( x = -2 \) or \( x = 2 \)
Inequality Symbols

$< \text{Less than } \Rightarrow -$ \text{ " left right " }$
$> \text{Greater than } $\Rightarrow$n$
\leq \text{Less than or equal to } \Rightarrow $\text{ left right “ }$
\geq \text{Greater than or equal to } $\Rightarrow$n$

Notice that the less than symbol looks like an “L”

Translations

$x \leq 5$

Graph:

Verbal: \text{ “}x \text{ is less than or equal to } 5\text{”} \text{ or “}x \text{ is at most } 5\text{”}

Translations

$|x| \leq 5$

Graph:

Verbal: \text{ “}x \text{ is less than or equal to } 5\text{”} \text{ and “}x \text{ is greater that } -5\text{”}

|ax + b| \leq c \quad ax + b < c \text{ and } ax + b \geq -c
|ax + b| \leq c \quad ax + b \leq c \text{ and } ax + b \geq -c
|ax + b| > c \quad ax + b < c \text{ or } ax + b > -c
|ax + b| \geq c \quad ax + b \leq c \text{ or } ax + b \geq -c

LESS = AND \text{,}\ \text{ GREATER = OR (Great OR, Less AND)}

Solve the following inequality

$2|x - 5| < 10$

Step 1: Isolate the absolute value.

$2|x - 5| < 10$

Step 2: Set up 2 inequalities.

$x - 5 < 5 \text{ and } x - 5 > -5$

$x < 10 \text{ and } x > 0$

Graph:

Verbal: \text{ “}x \text{ is less than 10 or } x \text{ is greater than 0}\text{”}

Set builder: $\{x | 0 < x < 10\}$

Solve the following inequality

$-2 \left| \frac{1}{2} x - 5 \right| - 1 < -13$

Graphically:

Verbal: \text{ “}x \text{ is greater than } 22 \text{ or } x \text{ is less than } -2\text{”}

Set Builder Notation: $\{x | x > 22 \text{ or } x < -2\}$
Solve the following inequality:

\[ 2 - 5 \left| \frac{1}{2} x + 1 \right| \geq -13 \]

\[ -5 \quad \frac{1}{2} x + 1 \geq -3 \]
\[ \frac{1}{2} x \leq -2 \]
\[ x \leq -4 \]

\[ \frac{1}{2} x + 1 \leq -3 \]
\[ \frac{1}{2} x \leq -4 \]
\[ x \leq -8 \]

Graphically:

Verbal: \( x \) is greater than or equal to 0 and less than or equal to 4.

Set Builder Notation: \( \{ x \mid -8 \leq x \leq 4 \} \)

Special Case #1:

\[ \left| \frac{1}{3} x - \frac{2}{5} \right| \geq -\frac{5}{2} \]

Note: All numbers that come out of the absolute value symbol are positive. \textbf{NO ANSWER CAN BE NEGATIVE!!!!!!!} Therefore the answer is "No Solution".

Special Case #2:

\[ \left| \frac{1}{3} x - \frac{2}{5} \right| \geq -\frac{5}{3} \]

Note: All numbers that come out of the absolute value symbol are positive. \textbf{EVERY ANSWER IS GREATER THAN OR EQUAL TO 5!!!!!!} Therefore the answer is "All Real Numbers".

Find the error(s)

\[ 4| x - 2 | < 20 \]
\[ | x - 2 | < 5 \]
\[ x - 2 < 5 \quad \text{or} \quad x - 2 > -5 \]

\[ x < 7 \quad \text{or} \quad x > -3 \]

Verbal: \( x \) is less than 7 or greater than -3.

Set: \( x < 7 \quad \text{or} \quad x > -3 \)
Graph of \( y = |x| \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
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<tbody>
<tr>
<td>-4</td>
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If \( a \) is positive the absolute value equation opens up. Ex: \( y = |x| \)

Graph of \( y = -|x| \)

<table>
<thead>
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<tbody>
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<td>-4</td>
</tr>
</tbody>
</table>

If \( a \) is negative the absolute value equation opens down. Ex: \( y = -|x| \)

Graphs of two variable absolute value equations

\[ y = a|x - h| + k \]

\( a \) determines how wide the "v" is in the absolute value equation as well as if the "v" is right side up or upside down.

The vertex of the absolute value equation is \((h, k)\). Notice that we must flip the sign of the number inside the absolute value equation to obtain the \( x \) component of the vertex.

Graph:

\[ y = \frac{1}{2}|x - 2| - 1 \]

\( a = \frac{1}{2} \)

Vertex:

\( (2, -1) \)

Slope to the right of vertex?

\( \frac{1}{2} \)

Slope to the left of the vertex?

\(- \frac{1}{2} \)

Axis of symmetry?

\( X \) value of the vertex or \( x = h \)

\[ x = \frac{2}{1} = 2 \]

Vertex max or min?

min

Is the point \((2, 5)\) a solution?

\( \text{NO} \)

Is the point \((3, 2)\) a solution?

\( \text{Yes} \)

\( X \) intercept?

\( y = 0 \)

\( 2, 0 \)

\( 5, 0 \)

\( Y \) intercept?

\( x = 0 \)

\( 0, -1 \)
Algebra 2

Lesson 1.3

| $y = -\frac{1}{3}|x| + 2$ | $y = -2|x + \frac{1}{3}|$ |
|-------------------------|-------------------------|
| $a = -\frac{1}{3}$     | $a = -2$             |

Verteex: $-\frac{1}{3}$
Slope to the right of vertex?
X intercept(s)? $(-6, 0), (4, 0)$
Y intercept? $0, 2$

X intercept? $-6, 0$
Y intercept? $-6, 0$

Given: $y = 2|x - 3| - 4$

Equation: $y = \frac{-2}{3}|x - 3|$

Find the $x_{int}$ algebraically:

\[
\frac{1}{-3 - 3} = y = 0 \\
\frac{1}{-3} = 2
\]

\[
x - 3 = 2 \quad x = 5
\]

Find the $y_{int}$ algebraically:

\[
y = 2|0 - 3| - 4 = 0, 2
\]

Find the $x_{int}$ algebraically:

\[
\frac{1}{3}|x + \frac{3}{1}| + 5 = 0
\]
\[
\frac{1}{3}|x + \frac{3}{1}| = -4 \quad \text{none}
\]

Find the $y_{int}$ algebraically:

\[
\frac{1}{3}|0 + \frac{3}{1}| + 4 = 0, 5
\]

Write the absolute value equation that has a right hand slope (dilation) of $\frac{2}{5}$, opens down and has a vertex of $(-3, -8)$.

\[
y = -\frac{2}{5}|x + 3| - 8
\]
Lesson 1.4

Quiz Next Class

Covers everything we have learned about absolute value functions so far and what we will learn today!

Anticipatory Set

What is the difference between the equation \( y = |x| \) and \( y = |x - 2| \)?

[Graph showing a V-shaped graph shifted right by 2 units]

How does the graph of \( y = |x + 5| \) relate to the graph of \( y = |x| \)?

[Graph showing a V-shaped graph shifted left by 5 units]

What is the difference between the equation \( y = |x| \) and \( y = \frac{2}{3} |x| \)?

[Graph showing a V-shaped graph with different dilation]

Vocabulary Transformations

Translation – vertical or horizontal shift
Vertical dilation – The graph is stretched or shrunk by a multiplied factor.
Reflection – The graph is flipped upside-down.

Describe all the transformations from the parent function \( f(x) = |x| \) to \( y = |x + 4| - 5 \)

Vertex 1 (0, 0)

Vertex 2 (-4, -5)

\( a_1 = \) \[
\begin{array}{c}
\text{opens up/down} \\
\text{down}
\end{array}
\]

\( a_2 = \) \[
\begin{array}{c}
\text{opens up/down} \\
\text{up}
\end{array}
\]

Translation: Graph is translated:
right left none \[
\begin{array}{c}
\text{up/down none} \\
\text{distance}
\end{array}
\]

Vertical dilation stretch/shrink none

[Diagram showing transformations]

Order of Transformations

First describe any horizontal translations (shifts right or left)
Second describe any vertical translations (shifts up or down)
Third describe any vertical dilations (stretches or shrinks)
Fourth describe the reflection (if necessary) (flips)

Describe all the transformations from the parent function \( f(x) = |x| \) to \( y = 5|x| + 1 \)

[Diagram showing transformations]

\[ \text{up 1} \]

\[ \text{dilation 5} \]
Lesson 1.4

Name:

HINT: All you need to plot is the vertex!!!!!!

Describe all the transformations from the pre-image \( y = |x + 3| - 1 \) to the image \( y = 3|x| + 2 \)

Vertex 1 \( (3, -1) \)

Vertex 2 \( (0, 2) \)

\( a_1 = \frac{1}{3} \) opens up

\( a_2 = 3 \) opens up

Translation: Graph is translated:

Right, left: none

Distance: \( \frac{3}{2} \) up, down: none

Vertical dilation: stretch/shrink: none

Reflection up/down: none

Describe all the transformations from the pre-image \( y = |x + 3| - 2 \) to the image \( y = \frac{1}{3}|x + 3| + 5 \)

\( (3, -2) \rightarrow (-3, 5) \)

Horizontal \( a_1 = 0 \)

Up 7

Reflected

Dilation \( \frac{1}{3} \)

Give a function which has a dilation of factor \( 7 \) and moves 9 units to the right from the function \( y = |x| \)

\( y = a|x - h| + k \)

\( h, k = 9, 0 \)

Substitute

\( y = \frac{7}{x - 9} \)

Give a function which moves 7 units up from \( y = |x| \)

\( y = |x| + 7 \)

POU: Write the inequality that represents the graph at right:

\( y > -2|x - 2| - 5 \)
I can add, subtract and multiply polynomials.

### Adding, Subtracting and Factoring Polynomials

Adding and subtracting polynomials

- Distribute if necessary
- Combine Like Terms

**Like Terms** have the same variables and the same exponent.

To minimize mistakes line the like terms underneath on another.

2 methods of Multiplying Polynomials:

1. **FOIL.** (First, Outside, Inside, Last) – only works for Binomials (2 terms)

2. **Box.** Works for all polynomials.

3. **Remember** - $(x - 5)^2 = (x - 5)(x - 5)$

**DO NOT SQUARE THE FIRST AND LAST TERMS!!!!!!**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> $(2x^2 - x + 3) + (3x^2 + 2x - 5)$</td>
<td><img src="image" alt="Addition" /></td>
</tr>
<tr>
<td><strong>2.</strong> $(3x^3 - 4x^2 + 3x - 7) - (x^3 + 2x^2 - 7x + 10)$</td>
<td><img src="image" alt="Subtraction" /></td>
</tr>
<tr>
<td><strong>3.</strong> $(x - 1)(x + 2)$</td>
<td><img src="image" alt="Multiplication" /></td>
</tr>
<tr>
<td><strong>4.</strong> $(2x - 4)(3x + 1)$</td>
<td><img src="image" alt="Multiplication" /></td>
</tr>
<tr>
<td><strong>5.</strong> $(2x - 1)^2$</td>
<td><img src="image" alt="Multiplication" /></td>
</tr>
<tr>
<td><strong>6.</strong> $(3x - 2)(x^3 - 7x + 1)$</td>
<td><img src="image" alt="Multiplication" /></td>
</tr>
</tbody>
</table>
I can factor Quadratics by splitting the middle term.

Factoring Polynomials by splitting the middle term.

Standard form of a trinomial $ax^2 + bx + c$

$a$ and $b$ are coefficients, $c$ is a constant and $x$ is the variable.

To factor trinomials, we will use the method known as splitting the middle term.

Factor: $6x^2 + 28x - 10$

1. Find the GCF first.

   $\frac{6x^2 + 28x - 10}{2}$

   $\frac{3x^2 + 14x - 5}{2}$

   $\frac{3x^2 + 15x - x - 5}{2}$

   $\frac{(3x^2 + 15x)(-x - 5)}{2}$

2. Find the values of $a$ and $c$.

   $a = \frac{3}{2}$ and $c = -\frac{10}{2}$

   Multiply $a$ and $c$: $a \cdot c = -\frac{30}{2}$

3. Split the middle term into 2 terms $14x = 15x - x$

   $3x^2 + 14x - 5$
   $3x^2 + 15x - x - 5$
   $(3x^2 + 15x)(-x - 5)$

4. Factor the GCF out of each set of parenthesis.

   $(3x^2 + 15x)(-x - 5)$
   $3x(x + 5) - 1(x + 5)$

5. Notice that there is a common factor of $(x + 5)$. We can now pull the common factor $(x + 5)$ out and write what is left in the second parenthesis.

   $3x(x + 5) - 1(x + 5)$
   $(x + 5)(3x - 1)$

6. Ensure the GCF is put back in front.

7. $3(x + 5)(3x - 1)$ is the factored form of $6x^2 + 28x - 10$.

   Factor: $12x^2 + 26x - 10$

   $\frac{12x^2 + 26x - 10}{2}$

   $\frac{6x^2 + 13x - 5}{2}$

   $\frac{(6x^2 + 15x)(-2x - 5)}{2}$

   $3x(2x + 5) - 1(2x - 5)$

   $2(3x - 1)(2x + 5)$

   Factor: $10x^2 - 34x - 24$

   $\frac{10x^2 - 34x - 24}{2}$

   $\frac{5x^2 - 17x - 12}{2}$

   $\frac{(5x^2 - 20x + 3x - 12)}{2}$

   $5x(x - 4) + 3(x - 4)$

   $2(5x - 3)(x - 4)$

   Factor: $9x^2 + 30x - 24$

   $\frac{9x^2 + 30x - 24}{3}$

   $\frac{3x^2 + 10x - 8}{3}$

   $\frac{(3x^2 + 12x - 2x - 8)}{3}$

   $3x(x + 4) - 2(x - 4)$

   $3(3x - 2)(x + 4)$
Learning Target: I can Graph Quadratic Equations with 2 Variables

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
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<tbody>
<tr>
<td>-3</td>
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<td>3</td>
<td>9</td>
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</table>

Vertex Form of the Quadratic Equation

\[ y = a(x - h)^2 + k \]

- \( h, k \) = Vertex
- \( a > 0 \) opens up
- \( a < 0 \) opens down (reflected)

\[ a > 1 \] shrink
\[ a < 1 \] stretch

Axis of symmetry \( x = h \)
Reflection of \( y_{\text{int}} \) \((2h, y_{\text{int}})\)
Real Roots cross \( x\)-axis, 1 or 2
Imaginary roots do not cross \( x\)-axis. Always 2

\[ y = a|x - h| + k \]

\[ y = x^2 \]

Similarities:

- \( a > 0 \) opens up
- \( a < 0 \) opens down
- \( h, k \) = vertex
- \( y_{\text{int}} \to x > 0 \)
- \( x_{\text{int}} \) set \( y \) to zero

Differences:
Graph \( y = -2(x - 2)^2 + 8 \) Opens up/down

\[ h, k = \quad 2, 4 \]

Axis of Symmetry:

\[ x = 2 \]

\[ y_{\text{int}} = (0, 6) \]

Reflection of \( y_{\text{int}} \) 4, 0

\[ x_{\text{int}} = -2(x - 2)^2 + 8 = 0 \]

\[ -2(x - 2)^2 = -8 \]

\[ (x - 2)^2 = 4 \]

\[ x - 2 = \pm 2 \]

\[ x = 4 \quad \text{or} \quad x = 0 \]

Roots: Real or Imaginary? \textbf{real} How Many? \textbf{2}

Vertex a max or min? \textbf{max}

Graph \( y = x^2 + 1 \) Opens up/down

\[ h, k = (0, 1) \]

Axis of Symmetry:

\[ x = 0 \]

\[ y_{\text{int}} = 0, 1 \]

Reflection of \( y_{\text{int}} \) 0, 1

\[ x_{\text{int}} = x^2 + 1 = 0 \]

\[ x^2 = -1 \]

\[ x = \sqrt{-1} \]

\[ \text{None} \]

Roots: Real or Imaginary? How Many?

Vertex a max or min?

Performance of Understanding

Write an equation for the graph in \( h, k \) form.

\[ u = a(x - h) + k \]

\[ = -3(x - 3) + 4 \]

Write the equation in standard \( y = ax^2 + bx + c \) form.
Solving Quadratic Equations

To solve quadratic equations in the h, k form, we isolate and separate.

1. Isolate the \((x - h)^2\)
2. Square Root both sides
3. Split into 2 equations "+" constant and "-" constant.
4. Solve for \(x\).

Solve \((x - 2)^2 - 6 = 0\)

1. Isolate \((x - 2)^2\)

\[
(x - 2)^2 = 42
\]

2. Square root both sides.

\[
\begin{align*}
(x - 2) &= \pm \sqrt{42} \\
 x - 2 &= \pm \frac{\sqrt{42}}{\sqrt{2}} \\
 x &= 2 \pm \sqrt{6}
\end{align*}
\]

3. Split into 2 equations

\[
\begin{align*}
 x - 2 &= \sqrt{6} \\
 x &= 2 + \sqrt{6} \\
 x &= 2 - \sqrt{6}
\end{align*}
\]

4. Solve

\[
\begin{align*}
 x &= 2 + \sqrt{6} \\
 x &= 2 - \sqrt{6}
\end{align*}
\]

The square root of \(\sqrt{-1} = i\).

\[
\begin{align*}
\sqrt{-4} &= 2i \\
\sqrt{-75} &= 5i\sqrt{3} \\
\sqrt{-9} &= 3i \\
\sqrt{-10} &= \sqrt{10}i \\
\sqrt{-25} &= 5i \\
\sqrt{-36} &= 6i
\end{align*}
\]

\(i\) Acts like a variable for like terms. To add, the number must have an \(i\). 

\[
3 + \sqrt{-4} = 2 + 2i
\]
Graphing Perfect Cube Polynomials

\[ y = a(x - h)^3 + k \]

Graph:

\[ y = \frac{1}{2}(x - 2)^3 + 4 \]

1. Find the inflection point \((h, k) = \left(\frac{2}{2}, \frac{4}{2}\right)\)

2. Find \(a = \sqrt{2}\)

   a. + means uphill left to right
   b. − means downhill left to right.

3. Find \(y_{\text{int}} = \frac{1}{2}(-2)^3 + 4\)

   \[ \frac{1}{2}(-8) + 4 = 0 \]

4. Find the \(x_{\text{int}} = \)

   \[ \frac{1}{2}(x-2)^3 = -4 \]

   \[ (x-2)^3 = -8 \]

   \[ x - 2 = -2 \]

   \[ x = 0 \]

   \[ \text{Domain: All real } \#	ext{s} \]

   \[ \text{Range: All real } \#	ext{s} \]

Graph:

\[ f(x) = -2(x + 1)^3 + 54 \]

\[ (-1, 54) \]

\[ -2(-1)^3 + 54 = 0, 52 \]

\[ -2(x+1)^3 = -8 \]

\[ (x+1)^3 = 4 \]

\[ x + 1 = 3 \]

\[ x = 2 \]

\[ \text{Domain: All real } \#	ext{s} \]

\[ \text{Range: All real } \#	ext{s} \]

POU: Solve for \(x\)

\[ 5(x - 3)^2 + 2x = 2(x - 3)^2 + 2x + 75 \]
The degree of a polynomial is the highest exponent in the expression.

<table>
<thead>
<tr>
<th>In ((h,k)) form</th>
<th>(y = 3(x - 2)^4 - 6)</th>
<th>Degree = 4</th>
<th>(y = 3x^4 - 2x^3 + 2x - 1)</th>
<th>Degree = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(y = 2(x - 1)^2 + 3)</td>
<td>Degree = 2</td>
<td>(y = x^2 - 1)</td>
<td>Degree = 2</td>
</tr>
<tr>
<td></td>
<td>(y = (x - 4)^{10} + 1)</td>
<td>Degree = 10</td>
<td>(y = -3x^2 + 2x^3 - 8x + 4)</td>
<td>Degree = 3</td>
</tr>
</tbody>
</table>

The maximum number of turns of a polynomial is equal to the degree minus 1.

<table>
<thead>
<tr>
<th></th>
<th>(y = x^4 - 5x + 3)</th>
<th>Number of Turns = 4 - 1 = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(y = -2(x - 4)^3 - 1)</td>
<td>Number of Turns = 3 - 1 = 2</td>
</tr>
<tr>
<td></td>
<td>(y = 3x^3 - 4x^5 + 10x^{17})</td>
<td>Number of Turns = 17 - 1 = 16</td>
</tr>
</tbody>
</table>

The total number of real, non-real (imaginary), and repeated roots of a polynomial is equal to the degree of the polynomial.

<table>
<thead>
<tr>
<th></th>
<th>(y = x^4 - 5x + 3)</th>
<th>Number of Roots = (\sqrt{4}) = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(y = -2(x - 4)^3 - 1)</td>
<td>Number of Roots = (\frac{3}{2})</td>
</tr>
<tr>
<td></td>
<td>(y = 3x^3 - 4x^5 + 10x^{17})</td>
<td>Number of Roots = (\frac{17}{2})</td>
</tr>
</tbody>
</table>

The leading coefficient is the coefficient of the term with the highest degree or the "a" in \((H,K)\) form.

<table>
<thead>
<tr>
<th></th>
<th>(y = x^3 + 3x - 5)</th>
<th>(a = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(y = -3x^{10} - 2x + 4)</td>
<td>(a = -3)</td>
</tr>
<tr>
<td></td>
<td>(y = -3x + 10x^3 - 4 + x^2)</td>
<td>(a = 10)</td>
</tr>
<tr>
<td></td>
<td>(y = 9(x - 7)^2 - 10)</td>
<td>(a = 9)</td>
</tr>
<tr>
<td></td>
<td>(y = 14 - 3(x - 5)^2)</td>
<td>(a = -3)</td>
</tr>
</tbody>
</table>

The leading coefficient "a" is (positive/negative) **positive**

because: \(RHS \uparrow\)

As \(x \to \infty\), \(y \to \infty\)

As \(x \to -\infty\), \(y \to -\infty\)

Give the coordinates of the x-intercepts: \( (5, 0), (2, 0), (-1, 0) \)

Give the coordinates of the y intercept: \( (0, 3) \)

The degree is **even**

Because: \(2 \text{ turns} / \text{opposite end behavior}\)

Total number of roots = \(3\)

The Roots are real/imaginary.

Because: \( \text{cross } x\text{-axis} \)

Domain: \( \mathbb{R} \) Range: \( \mathbb{R} \)
The leading coefficient “a” is positive because: \( R \) \( H \) S \( \downarrow \)

As \( x \to \infty \), \( y \to -\infty \)

As \( x \to -\infty \), \( y \to +\infty \)

Give the coordinates of the x-intercepts: \((-2, 0), (0, 0), (1, 0), (3, 0)\)

Give the coordinates of the y-intercept: \((0, 0)\)

The degree is even. Because: **opposite end behavior**

Total number of roots = 5

The Roots are real/imaginary. 
Because: **cross x - axis**

Domain: \( \mathbb{R} \) Range: \( \mathbb{R} \)

A) The value of the degree is 2

B) The total number of roots is 2

C) Describe the roots: Real/Imaginary Number: 2

D) The maximum number of turns is 1

E) The graph has **same** (opposite) right and left end behavior. Because: 2nd degree

F) As \( x \to \infty \), \( y \to -\infty \)

As \( x \to -\infty \), \( y \to +\infty \)

Domain: \( \mathbb{R} \) Range: \( y \leq -4 \)

The value of the degree is 3

The total number of roots is 3

Describe the roots 1 real 2 imaginary

The maximum number of turns is 2

The graph has **opposite** right and left end behavior. Because: 3rd degree

As \( x \to \infty \), \( y \to -\infty \)

As \( x \to -\infty \), \( y \to +\infty \)

Domain: \( \mathbb{R} \) Range: \( \mathbb{R} \)
1. Find the root
2. Set up synthetic division
3. Multiply and Combine
4. Take care of the remainder

**Synthetic Division**

1. $(2x^3 - 5x^2 + 3x + 7) ÷ (x - 2)$
   
   **Root of $x - 2$ is $x = 2$**

   **Coefficients of $2x^3 - 5x^2 + 3x + 7$**
   
   $\begin{array}{cccc}
   2 & -5 & 3 & 7 \\
   \downarrow & & & \\
   2 & -1 & 1 & 9 \\
   \end{array}$

   **$x^2 - x + 1 + \frac{9}{x - 2}$**

2. $(x^4 - 10x^2 - 2x + 4) ÷ (x + 3)$

   -2 | 3 8 5 -7
   -2 | -2 -4 -2
   \hline
   3 2 1 -9
   $3x^2 + 2x + 1 + \frac{-9}{x + 2}$

3. $(2x^3 + 7x^2 - 4x^2 - 27x - 18) ÷ (x - 2)$

   -2 | 2 7 -4 -27 -15
   -4 | 22 36 14
   \hline
   2 11 16 9 0
   $2x^3 + 11x^2 + 18x + 9$
We are going to use the Zero Product Property to solve Quadratic Equations.

We are going to use the Zero Product Property to solve Quadratic Equations.

Anticipatory Set:
What values will make the following statement true?

\[(x + 7)(2x - 3) = 0\]

\[x + 7 = 0 \quad 2x - 3 = 0\]

Solve for x.

\[x = -7 \quad 2x = 3\]

\[x = \frac{3}{2}\]

If \(x = -7\) then \((-7 + 7)(2x - 3) = 0\)

\[(0)(2x - 3) = 0\]

If \(x = \frac{3}{2}\) then

\[(x + 7)(2 \frac{3}{2} - 3) = 0\]

\[(x + 7)(3 - 3) = 0\]

\[(x + 7)(0) = 0\]

What values will make the following statements true?

\[2x(3x - 7)(x - 4) = 0\]

\[2x = 0 \quad 3x - 7 = 0 \quad x - 4 = 0\]

\[x = 0 \quad x = \frac{7}{3} \quad x = 4\]

\[3(x - 1)(x + 4) = 0\]

\[x - 1 = 0 \quad x + 4 = 0\]

\[x = 1 \quad x = -4\]

In the above examples, the Polynomials where already factored for us. Now we will factor and then solve the Polynomial.

Step 1: Factor

\[2x^2 - 14x - 60 = 0\]

\[2(x^2 - 7x - 30)\]

\[2(x - 10)(x + 3)\]

Step 2: Set each factor equal to zero.

\[x = 10 \quad x = -3\]

\[10x^2 - 5x - 15 = 0\]

\[5(2x^2 - x - 3)\]

\[5(2x - 3)(x + 1)\]

\[x = \frac{3}{2} \quad x = -1\]
Given:
\( y = x^2 - 29x + 100 \)
Find: \( y_{\text{int}}: (0, 100) \)
Find: \( x_{\text{int}}: (25, 0), (4, 0) \)
\((x - 25)(x - 4)\)

Given:
\( y = -(x - 3)^2 + 100 \)
Find: \( y_{\text{int}}: (0, 91) \)
Find: \( x_{\text{int}}: (13, 0), (-7, 0) \)
\( x - 3 = 10 \quad x - 3 = -10 \)
\( x = 13 \quad x = -7 \)
Find the $x_m$ of $y = x^3 + 4x^2 - 16x - 64$

- X-intercepts
  \[
  (x^3 + 4x^2)(-16x - 64) \\
  x^2(x + 4) - 16(x + 4) \\
  (x^2 - 16)(x + 4) \\
  (x - 4)(x + 4) \\
  (-4, 0)(-4, 0)x = -4 \\
  \]
- $y_{inf} = (0, -64)$
- Max number of turns $2$

Lead Coefficient positive

Graph:

1. Plot the x - intercepts and y - intercepts
2. Plot end behavior
3. Put in number of the turns
4. Connect the graph.

$f(x) = -x^4 + 81x^2$

- X-intercepts
  \[
  -x^2(x^2 - 81) \\
  -x^2(x - 9)(x + 9) \\
  x = 0 \quad x = 0 \quad x = 9 \quad x = -9 \\
  \]

- The value of the leading coefficient "a" is positive or negative? Why? $\text{RHS} \uparrow$
- The y intercept("c") is $0 - 3$
- The degree must be at least 2 because 2 turns
- Give the x intercepts: $(-3, 0)x = 3, 0x = 1, 0$
- Give all possible factors
  \[
  (x + 3)(x + 3)(x - 1) \\
  \]

Write the equation in factored form.

\[
y = (x + 3)(x + 3)(x - 1) \\
\]
The max number of turns of a polynomial is the degree of the polynomial minus 1.

- If the leading coefficient is positive then the graph rises right.
- If the leading coefficient is negative then the graph falls right.
- If the degree is even the left end behavior matches the right end behavior.
- If the degree is odd the left end behavior is the opposite of the right end behavior.

\[ y = -x^3 - 6x^2 - 9x \]

- **X-intercepts**
  \[ x \left( x^2 + 6x + 9 \right) \]
  \[ x (x + 3)(x - 3) \]
  \[ x = 0, \quad x = -3, \quad x = \pm 3 \]

- **y-intercept**
  \[ \frac{2}{\infty}, \quad y \to -\infty \]

- **Limit**
  \[ x \to \infty, \quad y \to \infty \]

- **Coeficient** negative

\[ y = x^3 - 4x^2 - 9x + 36 \]

The process of factoring a polynomial with 4 terms is the same as factoring a Quadratic by splitting the middle term.

Consider the middle term to be already split

\[
\frac{(x^3 - 4x^2)(-9x + 36)}{x^2 - 9} = 0
\]

Find the GCF of each set of parenthesis

\[ x^2(x - 4) - 9(x - 4) = 0 \]

\[ (x^2 - 9)(x - 4) = 0 \]

\[ (x - 3)(x + 3)(x - 4) = 0 \]

\[ x = 3, -3, 4 \]
Lesson 3.3

Quadratic Formula

Used to find the roots when the Quadratic is not factorable.

PEMDAS!!!!!!!

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ a, b, & c \text{ are from } y = ax^2 + bx + c \]

Find the \( x_{\text{int}} \) of \( f(x) = x^2 - 4x + 2 \)

\[ a = 1 \]
\[ b = -4 \]
\[ c = 2 \]

\[ x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} \]
\[ = \frac{-(-4) \pm \sqrt{16 - 8}}{2} \]
\[ = \frac{-(-4) \pm 2\sqrt{2}}{2} \]
\[ = -1 \pm \sqrt{2} \]

Real/imaginary

Find the roots of \( y = x^2 - 6x + 4 \)

\[ c = \sqrt{36 - 4(1)(4)} \]
\[ \frac{6 \pm \sqrt{16}}{2(1)} \]
\[ \frac{6 \pm 4}{2} \]
\[ 3 \pm 2 \sqrt{2} \]

Real/imaginary

Find the roots of \( y = -2x^2 - 5x + 3 \)

\[ 5 \pm \sqrt{25 - 4(-2)(3)} \]
\[ \frac{5 \pm \sqrt{25 + 24}}{2} \]
\[ \frac{5 \pm 7}{2} \]
\[ 3, -1 \]

Real/imaginary

Converting from Standard Form \( y = ax^2 + bx + c \) to Vertex Form \( y = a(x-h)^2 + k \)

To convert from Standard for to Vertex or \( h, k \) form, we must find the value of \( h \).

To find the axis of symmetry we use the formula \( x = \frac{-b}{2a} \).

Note that this is the first part of the equation

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Convert \( y = x^2 + 6x - 1 \) to vertex form.

\[ a = 1 \]
\[ b = 6 \]
\[ c = -1 \]

\[ x = \frac{(-6)}{2(1)} = -3 \]

The axis of symmetry of the Quadratic is \( x = -3 \). The axis of symmetry crosses through the vertex. Therefore \( x = h \). \( x = h = -3 \).

To find \( k \), plug \( x = h = -3 \) back into the original equation \( y = x^2 + 6x - 1 \).

\[ k = x \left( -3 \right)^2 + 6\left( -3 \right) - 1 = -10 \]

\( h, k = -3, -10 \)

\[ y = a(x-h)^2 + k \]

\[ y = (x+3)^2 - 10 \]
Convert $y = x^2 - 4x + 2$ to $y = a(x - h)^2 + k$

Find $h$
\[ \frac{4}{2} = 2 \]
\[ a = 1 \]

Find $k$
\[ (2)^2 - 4(2) + 2 \]
\[ y - 4 + 2 = -2 \]

Write in vertex or $h, k$ form.
\[ y = (x - 2)^2 - 2 \]

Convert $y = -x^2 + 6x + 9$ to $h, k$ form.

\[ \frac{-6}{2} = -3 \quad a = -1 \]

\[ -(-3)^2 + 6(-3) + 9 \]
\[ -9 - 18 + 9 \]
\[ -18 \]

\[ y = -(x + 3)^2 - 18 \]

The Discriminant $b^2 - 4ac$ determines whether the Quadratic has real or imaginary roots. Notice the discriminant is also a part of the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

If:
- $b^2 - 4ac > 0$ then 2 real roots
- $b^2 - 4ac = 0$ then 1 real root
- $b^2 - 4ac < 0$ then 2 non-real roots

Use the Discriminant to determine the number and nature of the roots for the Quadratic $y = x^2 + 6x + 8$

\[ a = 1, \quad b = 6, \quad c = 8 \]

\[ (6)^2 - 4(1)(8) = 4, \quad \text{which is } > 0. \quad 2 \text{ Real roots} \]

\[ 36 - 32 = 4 \]

Use the Discriminant to determine the number and nature of the roots for the Quadratics:

\[ y = x^2 - 8x + 10 \]

\[ 64 - 4(1)(10) \]
\[ 64 - 40 > 0 \]
\[ 2 \text{ real} \]

\[ y = 2x^2 - 8x + 8 \]

\[ 64 - 4(2)(16) \]
\[ 64 - 64 = 0 \]
\[ 1 \text{ real} \]

\[ y = x^2 - 7x + 15 \]

\[ 49 - 4(1)(15) \]
\[ 49 - 60 < 0 \]
\[ 2 \text{ imaginary} \]
### I can factor and graph a difference of 2 cubes polynomial

**Sum or Difference of 2 Perfect Cubes**

\[
\begin{align*}
    a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\
    a^3 + b^3 &= (a + b)(a^2 - ab + b^2)
\end{align*}
\]

#### Factor \( y = 8x^3 + 27 \)

1st: Identify \( a \) & \( b \).

\[
\begin{align*}
    a^3 &= 8x^3 \\
    b^3 &= 27 \\
    \sqrt[3]{a^3} &= \sqrt[3]{8x^3} \\
    \sqrt[3]{b^3} &= \sqrt[3]{27} \\
    a &= 2x \\
    b &= 3
\end{align*}
\]

\[
\begin{align*}
    a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \\
    8x^3 + 27 &= (2x + 3)(4x^2 - 2x \cdot 3 + 3^2) \\
    8x^3 + 27 &= (2x + 3)(4x^2 - 6x + 9)
\end{align*}
\]

**Sketch:**

For all sum and difference of 2 perfect cubes, they will have 1 real and 2 imaginary (non-real) roots.

#### Factor \( y = 64x^3 - 1 \)

\[
\begin{align*}
    a &= 4x \\
    b &= 1 \\
    c &= 9
\end{align*}
\]

\[
\begin{align*}
    b^2 - 4ac &= 36 - 4(4)(9) \\
    &= -36 < 0 \\
    \text{no real roots}
\end{align*}
\]

#### Factor \( x^3 + 125y^3 \)

\[
(x + 5y)(x^2 + 5xy + 25y^2)
\]

\[
(x^2)
\]
### Using the TI Calculator to examine polynomials

**Given:** \( y = -x^2 + 8x + 7 \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>Set Window on the TI-83 to see all the turns:</td>
</tr>
<tr>
<td>( X_{\text{min}} = )</td>
<td></td>
</tr>
<tr>
<td>( X_{\text{max}} = )</td>
<td></td>
</tr>
<tr>
<td>( Y_{\text{min}} = )</td>
<td></td>
</tr>
<tr>
<td>( Y_{\text{max}} = )</td>
<td></td>
</tr>
<tr>
<td>Find ( y_{\text{int}} = (<strong><strong>,</strong></strong>) )</td>
<td></td>
</tr>
<tr>
<td>(See Section A of the handout)</td>
<td></td>
</tr>
</tbody>
</table>

**Find the Vertex: (____,____)\)  
(See section B of the handout)

**Find the \( x_{\text{int}} = (____,____); (____,____) \) \)  
(See section C of the handout)

**If \( y = 2 \), Find the value(s) of \( x \). \)  
(See section D of the handout)

**When \( x = 11 \), \( y = _____ \) \)  
(See section E of the handout)

---

**Given:** \( y = x^3 + 2x^2 - 9x - 18 \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>Graph on the Calculator.</td>
</tr>
<tr>
<td>c.</td>
<td>Set Window on the TI-83 to see all the turns:</td>
</tr>
<tr>
<td>( X_{\text{min}} = )</td>
<td></td>
</tr>
<tr>
<td>( X_{\text{max}} = )</td>
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<td>( Y_{\text{min}} = )</td>
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<td></td>
</tr>
<tr>
<td>Find ( y_{\text{int}} = (<strong><strong>,</strong></strong>) )</td>
<td></td>
</tr>
<tr>
<td>(See Section A of the handout)</td>
<td></td>
</tr>
</tbody>
</table>

**Find the Relative Max: (____,____) \)  
(See section B of the handout)

**Find the Relative Min: (____,____) \)  
(See section B of the handout)

**Find the \( x_{\text{int}} = (____,____); (____,____) \) \)  
(See section C of the handout)

**If \( y = 2 \), Find the value(s) of \( x \). \)  
(See section D of the handout)

**When \( x = 11 \), \( y = _____ \) \)  
(See section E of the handout)
Lesson 3.5

<table>
<thead>
<tr>
<th>Power Rule</th>
<th>Multiply</th>
<th>Division</th>
<th>Multiplication &amp; Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parenthesis to a power</td>
<td>Add Exponents</td>
<td>Subtract Exponents</td>
<td>Multiply 1st, then divide</td>
</tr>
<tr>
<td>Multiply Exponents</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
(3x^4y)^3
\]

\[
27x^{12}y^3
\]

\[
x^{11} \cdot x^3
\]

\[
x^{14}
\]

\[
\frac{-6x^8y^3}{-4xy^{20}}
\]

\[
\frac{2x^3y}{x^6}
\]

\[
\frac{xy^8}{16}
\]

\[
\frac{2x^3y^7}{y^{17}}
\]

\[
\frac{3x^7}{2y^{17}}
\]

\[
\frac{7x^4y^3}{15x^6}
\]

\[
\frac{x^3y^3}{8}
\]

Review: *if \( \sqrt{-1} = i \), then \( i^2 = -1 \)

Multiplying Binomials means BOX!

Multiply: (Check using the calculator)

\[
(5-i)(3+2i)
\]

\[
\begin{array}{c|ccc}
5 & -i & 2i \\
3 & 15 & -3i \\
2i & 10 - 2i & -2i \\
\hline
15 & 10i & -3i + 2 \\
\end{array}
\]

\[
15 + 10i - 3i + 2 = 17 + 7i
\]

Multiply: (Check using the calculator)

\[
(7 + 2i)^3
\]

\[
\begin{array}{c|ccc}
7 + 2i & 9 + 4i & 14i \\
7 & 49 & 14i \\
2i & 4i & 4i \\
\hline
45 + 26i
\end{array}
\]

Synthetic Division

If the remainder is 0, then it is a root.

(-3, 0) is one root of the equation \( y = 3x^3 + 5x^2 - 21x - 27 \). Use synthetic division to find the other root(s).

\[
\begin{array}{c|cccc}
-3 & 3 & 5 & -21 & -27 \\
\hline
& -15 & 30 & 27 \\
3 & -10 & -9 \\
\end{array}
\]

Use synthetic division to prove that (1, 0) is a root of \( y = x^4 - 1 \).
Use synthetic division to prove that \((x - 2)\) is a factor of 
\[y = x^3 + 11x^2 - 4x - 44\]

\[
2 \overline{1 \quad 11 \quad -4 \quad -44}
\]
\[
\quad \quad 2 \quad 22 \quad 0
\]
\[
1 \quad 13 \quad 22 \quad \text{Remainder 0}
\]

\[x^2 + 13x + 22 \quad \text{ac} = 22\]
\[x^2 + 11x + 2x + 22\]
\[x(x+11) + 2(x+11)\]
\[(x+2)(x+11)(x-2)\]
\[\uparrow \text{factors}\]

POU:
Is \(x - 3\) a factor of \(f(x) = x^3 - 2x^2 - 23x + 60\)?

\[
3 \overline{1 \quad -2 \quad -23 \quad 60}
\]
\[
\quad \quad 3 \quad 3 \quad -60
\]
\[
1 \quad 1 \quad -20 \quad 0
\]
\[
\text{Roots: } x = 3, -5 + \sqrt{6}, -5 - \sqrt{6}
\]
\[x^2 + 5x - 4x + 20\]
\[x(x+5) - 4(x+5)\]
\[(x-4)(x+5)(x-3)\]
\[\downarrow \text{factors}\]
### Unit 3 Lesson 1

**ZERO PRODUCT PROPERTY**

If \( ab = 0 \) then \( a = 0 \) or \( b = 0 \)

We are going to use the Zero Product Property to solve Quadratic Equations.

<table>
<thead>
<tr>
<th>Anticipatory Set:</th>
<th>What values will make the following statements true?</th>
</tr>
</thead>
<tbody>
<tr>
<td>What values will make the following statement true?</td>
<td>( 2x(3x-7)(x-4) = 0 )</td>
</tr>
<tr>
<td>( (x+7)(2x-3) = 0 )</td>
<td>( x = 0 ) ( 3x-7 = 0 ) ( x-4 = 0 )</td>
</tr>
<tr>
<td>( x + 7 = 0 ) ( 2x - 3 = 0 )</td>
<td>( x = 0 ) ( x = \frac{7}{3} ) ( x = 4 )</td>
</tr>
</tbody>
</table>

Solve for \( x \).

\[
\begin{align*}
    x & = -7 \\
    2x & = 3 \\
    x & = \frac{3}{2}
\end{align*}
\]

If \( x = -7 \) then \( (x+7)(2x-3) = 0 \)

\[
\begin{align*}
    (0)(2x-3) & = 0 \\
    (x+7)(3-3) & = 0 \\
    (x+7)(0) & = 0
\end{align*}
\]

If \( \frac{3}{2} \) then

\[
\begin{align*}
    (x+7)(\frac{3}{2}-3) & = 0 \\
    (x+7)(3-3) & = 0 \\
    (x+7)(0) & = 0
\end{align*}
\]

In the above examples, the Polynomials where already factored for us. Now we will factor and then solve the Polynomial.

### Step 1: Factor

\[
2x^2 - 14x - 60 = 0
\]

\[
2(x^2 - 7x - 30) \\
2(x - 10)(x + 3)
\]

### Step 2: Set each factor equal to zero.

\[
\begin{align*}
    x & = 10 \\
    x & = -3
\end{align*}
\]

\[
10x^2 - 5x - 15 = 0
\]

\[
5(2x^2 - x - 3) \\
5(2x-3)(x+1)
\]

\[
\begin{align*}
    x & = \frac{3}{2} \\
    x & = -1
\end{align*}
\]

\[
\begin{align*}
    x & = 0 \\
    3x-7 & = 0 \\
    x-4 & = 0
\end{align*}
\]

\[
\begin{align*}
    x & = 0 \\
    x & = \frac{7}{3} \\
    x & = 4
\end{align*}
\]
\[3x^3 - 12x^2 - 63x = 0\]
\[3x(x^2 - 4x - 21)\]
\[3x(x-7)(x+3)\]
\(x = 0 \quad x = 7 \quad x = -3\]

\[4x^2 - 25 = 0\]
\[(2x - 5)(2x + 5)\]
\(x = \frac{5}{2} \quad x = -\frac{5}{2}\]

\[x^2 - 9 = 0\]
\(x = \pm 3\)

---

**Performance of Understanding**

**Given:**
\[y = x^2 - 29x + 100\]

Find: \(y_{\text{int}}: (0 , 100)\)

Find: \(x_{\text{int}}: (25 , 0), (4 , 0)\)

\((x - 25)(x - 4)\)

**Given:**
\[y = -(x - 3)^2 + 100\]

Find: \(y_{\text{int}}: (0 , 91)\)

Find: \(x_{\text{int}}: (13 , 0), (-7 , 0)\)

\(x - 3 = 10 \quad x - 3 = -10\)

\(x = 13 \quad x = -7\)
Rational Numbers: Whole Numbers and Fractions

Exponents: The number raised to.

If an exponent is a fraction, the denominator is the index of the radical.

\[ 4^{\frac{1}{2}} = \sqrt{4} = 2 \]

Always simplify the Radical FIRST!!!!!!!
Otherwise the numbers can get to large

\[ 8^{\frac{1}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4 \]

Evaluate the expression without using the Calculator.

\[ 27^{\frac{2}{3}} = \sqrt[3]{27} = 3 \quad (-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2 \]

\[ (-64)^{\frac{1}{3}} = \sqrt[3]{-64} = -4 \]

Rewrite the expression using exponent notation

\[ 4^{\frac{1}{2}} = 3^{\frac{1}{2}} \quad (\sqrt[3]{3})^3 = 3^{\frac{1}{3}} \]

\[ \frac{1}{(\sqrt[3]{7})^{3}} = \frac{1}{7^{\frac{3}{3}}} \]

Negative Exponents

Negative exponents change the location form numerator to denominator or denominator to numerator.

\[ 2^{-1} = \frac{1}{2} \]

\[ 4^{-\frac{1}{2}} = \frac{1}{\sqrt[2]{4}} = \frac{1}{2} \]

\[ \frac{1}{2}^{-1} = 2^1 = 2 \]

Evaluate the expression without using the calculator.

\[ (16)^{\frac{3}{2}} = \left(\sqrt[2]{16}\right)^3 = 2^4 = 16 \quad 2(25)^{\frac{1}{2}} = \frac{22}{\sqrt{25}} = \frac{2}{5} \]

\[ \frac{1}{27^{\frac{1}{3}}} = \left(\sqrt[3]{27}\right)^3 = 9 \]

Evaluating Radical Expressions and Radical Exponents using the Calculator

Evaluate \( 5^{\frac{1}{3}} \) using the calculator.

Type in \( 5^{\frac{1}{3}} \)

Evaluate \( \sqrt[3]{125} \)

Type in \( 125^{\frac{1}{3}} \)

Or press \[ \text{MATH} \]

Evaluate using a calculator (Round to 3 decimal place):

\[ 14^{\frac{2}{3}} = 1.172 \]

\[ (-103)^{\frac{2}{5}} = 6.385 \]

\[ \frac{1}{103^{\frac{1}{5}}} = 103^{-\frac{1}{5}} = -1.5 \]

Simplifying Radical and Rational Expressions

Simplify the expressions

\[ \sqrt[4]{64a^2b^3c^7} = 2a^{\frac{1}{2}}b^{\frac{3}{2}}c^{\frac{7}{4}} \]

\[ \sqrt[2]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c} \]

\[ 2abc \sqrt[4]{a^2c^3} \]

\[ \frac{20x^3}{y^6} = \frac{2x}{y^3} \sqrt{5x} \]

Simplify the expressions

\[ (8a^3b^4)^{\frac{1}{2}} = \sqrt[2]{8a^3b^4} \]

\[ 2ab^2 \left( 2a \right)^{\frac{1}{2}} \]
### Solving Radical Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\sqrt{x} + 2)^2 = 9)</td>
<td>(x + 2 = 3 \Rightarrow x = 1)</td>
</tr>
<tr>
<td>((\sqrt{x} - 7)^3 = 27)</td>
<td>(\sqrt{x} - 7 = 3 \Rightarrow x = 16)</td>
</tr>
<tr>
<td>((2x - 3)^3 = 4)</td>
<td>(2x - 3 = \sqrt[3]{4} \Rightarrow x = \frac{\sqrt[3]{4} + 3}{2})</td>
</tr>
<tr>
<td>((3x + 1)^3 = 8)</td>
<td>(3x + 1 = 2 \Rightarrow x = -\frac{1}{3})</td>
</tr>
</tbody>
</table>

#### Performance of Understanding

Find the x and y intercepts.

- **y-intercept:**
  \[ y = \sqrt{x + 4} - 3 \]

  - \[ \sqrt{x + 4} = 3 \]
  - \[ x + 4 = 9 \]
  - \[ x = 5 \]

  ![Intercept Graph](image)

- **x-intercept:**
  \[ \sqrt{x + y} = 3 \]

  - \[ \sqrt{y + 3} = 3 \]
  - \[ y + 3 = 9 \]
  - \[ y = 6 \]

  ![x-intercept Graph](image)
### Algebra 2

**Lesson 4.2**

**Definitions:** *Anything to the Zero power is 1.*

\[
x^0 = 1 \\
2^0 = 1 \\
25^0 = 1 \\
1,000,000,000^0 = 1
\]

**Why?**

\[
\frac{21^3}{21^3} = 1, \text{ anything divided by itself is 1.}
\]

When we divide we subtract exponents.

\[
21^{3-3} = 21^0 = 1
\]

---

**I can Multiply and Divide rational exponents with the Same Base**

- **Multiplying same base – add exponents.**
  \[
x^2 \cdot x^3 = x^{\frac{5}{3} + \frac{5}{3}} = \frac{5}{3} \cdot \frac{5}{3} = 5^2 = 25
  \]
- **Dividing the same base – subtract exponents.**
  \[
  \frac{2x^{\frac{1}{3}}}{18x^3y} = \frac{x^{\frac{1}{2}}}{9y^{\frac{1}{15}}}
  \]

---

**I can Multiply and Divide Radicals with the same Index**

If the index of the radicals are the same **make one big happy radical!!!**

\[
\sqrt[3]{5} \cdot \sqrt[3]{25} = \sqrt[3]{125} = 5
\]

\[
\frac{3\sqrt[3]{125}}{\sqrt[3]{25}} = \frac{3}{5}
\]

\[
\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} = 3
\]

\[
\frac{\sqrt{48}}{\sqrt{6}} = \frac{\sqrt{8}}{2} = \frac{2}{2} = 1
\]

---

**I can Add Subtract Radicals**

In order to subtract or add radicals the must have the same index and the same radicand.

- \[
  \sqrt{2\sqrt{x} + 3\sqrt{x}} = 5\sqrt{x}
  \]
  Can do!!!!!

- \[
  \sqrt{4\sqrt{2} + 2\sqrt{2}} = 6\sqrt{2}
  \]

**Simplify:**

\[
\sqrt[3]{16} + \sqrt[3]{54} + \sqrt[3]{24}
\]

1. Simplify each radical.
   \[
   \sqrt[3]{16} = 2\sqrt[3]{2}, \quad \sqrt[3]{54} = 3\sqrt[3]{2}, \quad \sqrt[3]{24} = 2\sqrt[3]{3}
   \]

2. Combine Like Terms.
   \[
   2\sqrt[3]{2} + 3\sqrt[3]{2} + 2\sqrt[3]{3} = 5\sqrt[3]{2} + 2\sqrt[3]{3}
   \]
\[
\begin{array}{c|c}
2\sqrt{x} + 2\sqrt{y} & \text{Simplify:} \\
x + x^\frac{1}{3} & \sqrt{-16} - \sqrt{1} \\
2\sqrt{2} + 3\sqrt{2} & 4i - i = 3i \\
\sqrt{x} + \sqrt{x} & 3\sqrt{3} - 5\sqrt{48} \\
\sqrt{5} + \sqrt{2} & 3\sqrt{3} - 20\sqrt{2} \\
\end{array}
\]

\[
\begin{array}{c|c}
4\sqrt{24} - 2\sqrt{3} & 3\sqrt{3} - 20\sqrt{2} \\
6\sqrt{3} - 2\sqrt{15} & -17\sqrt{3} \\
4\sqrt{3} & 7\sqrt{x} - 3\sqrt{x} = 4\sqrt{x} \\
\end{array}
\]

\[
\begin{array}{c|c}
4\sqrt{2} - 3\sqrt{50} & 4i\sqrt{2} - 15i\sqrt{2} \\
\end{array}
\]

\[
\begin{array}{c|c}
\end{array}
\]

\[
\begin{array}{c}
\text{Performance of Understanding} \\
\text{If you can do this problem you have a good understanding of the lesson!!!!!!}
\end{array}
\]

\[
\frac{5\sqrt{3} - 2\sqrt{3} + 7\sqrt{3}}{2\sqrt{2}} = \frac{10\sqrt{3}}{2\sqrt{2}} = \frac{5\sqrt{3}}{\sqrt{2}}
\]
Graphing Radical Equations $y = \sqrt{x}$

- **Graph of a square root equation**
  - $y = \sqrt{x}$

- **Fill in the table:**
<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sqrt{x}$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

- **Graph The following equation:**
  - $y = -(x - 3)^{\frac{1}{2}} + 2$
  - Give the Parent Equation: $y = x^{\frac{1}{2}}$
  - Describe the transformation:
    - Left: $3$
    - Right: __
    - Up/Down: __
    - Reflected: Yes/No

- **If possible give the X-intercept (y = 0)**
  - $-(x-3)^{\frac{1}{2}} + 2 = 0$
  - $x - 3 = 4$
  - $x = 7$

- **Y-intercept (x=0)**
  - $-(6-3)^{\frac{1}{2}} + 2 = 0$
  - No

- **Graph The following equation:**
  - $y = (x + 81)^{\frac{1}{2}} - 2$  
  - $(-81, -2)$
  - Give the Parent Equation: $y = x^{\frac{1}{2}}$
  - Describe the transformation:
    - Left: $5$
    - Right: __
    - Up/Down: __
    - Reflected: Yes/No

- **If possible, give the X-intercept (y = 0)**
  - $(x + 51)^{\frac{1}{2}} = 2$
  - $x + 51 = 4$
  - $x = -77$

- **Y-intercept (x=0)**
  - $(0 + 51)^{\frac{1}{2}} - 2$
  - $9 - 2 = 7$

- **Graphing Radical Equations $y = \sqrt[3]{x}$**

- **Graph of a cube root equation**
  - $y = \sqrt[3]{x}$

- **Fill in the table:**
<table>
<thead>
<tr>
<th>$x$</th>
<th>$\sqrt[3]{x}$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

- $(h, k)$ is the inflection point of the graph
Graph the following equation:

\[ y = (x - 8)^{\frac{1}{3}} + 1 \]

Give the Parent Equation: \( y = \sqrt[3]{x} \)

\( h, k = 8, -1 \)

Describe the transformation:

Left/Right: \( \bigcirc \) \( 8 \)

Up/Down: \( \bigcirc \) \( 1 \)

Reflected: Yes/No \( \bigcirc \)

If possible, give the

X-intercept (\( y = 0 \))

\( y = (0 - 8)^{\frac{1}{3}} + 1 \)

\( 8 - 8 = -1 \)

\( x = 7, 0 \)

\( \frac{1}{3} - 1 \)

\( y = -2 + 1 \)

\( = -1 \)

\( 0, -1 \)

---

I can determine the Radical Equation from a Graph

Describe the transformation from \( y = \sqrt[3]{x} \):

Left/Right: \( \bigcirc \) \( 2 \)

Up/Down: \( \bigcirc \) \( 0 \)

Reflected: Yes/No \( \bigcirc \)

\( y = \sqrt[3]{x - h + k} \)

\( y = \sqrt[3]{x - 2} + 1 \)

Describe the transformation from \( y = \sqrt[3]{x} \):

Left/Right: \( \bigcirc \) \( 2 \)

Up/Down: \( \bigcirc \) \( 1 \)

Reflected: Yes/No \( \bigcirc \)

\( y = \sqrt[3]{x - h + k} \)

\( y = -\sqrt[3]{x - 2} + 1 \)

POU: Write the correct equation for the graphs below:

\[ y = -\sqrt[3]{x^2 + 3} + 4 \]

\[ y = \frac{2}{\sqrt[3]{x^2} + 3} + 3 \]
Evaluate to 3 decimal places

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Evaluate $19^{-2/3}$</td>
<td>$140$</td>
</tr>
<tr>
<td>2. Evaluate $20^{1/5}$</td>
<td>$1.94$</td>
</tr>
<tr>
<td>3. Evaluate $\sqrt[4]{20}$</td>
<td>$2.1147$</td>
</tr>
<tr>
<td>4. Evaluate $\sqrt[4]{44}$</td>
<td>$1.717$</td>
</tr>
</tbody>
</table>

For 1995 through 2015 the amount spent in millions for shoe advertisement (in millions of dollars) in the US can be modeled by:

$$A = 0.56t^2 + 100$$

Where $t = 0$ represents 1995.

1. Determine how much money was spent in 2001.

$$A = 0.56(5)^2 + 100 = 149.36$$

2. Determine how much money was spent in 2004.

$$A = 0.56(9)^2 + 100 = 236.08$$

3. Determine which year $359$ million was spent

$$\frac{A - 359}{100} = 0.56t^2 + 100$$

$$1995 \rightarrow 2006$$

$$t = 11.6$$

4. Determine which year $410$ million was spent

$$\frac{410 - 359}{100} = 0.56t^2 + 100$$

$$310 = 0.56t^2 + 100$$

$$110 = 0.56t^2$$

$$t = 12.15$$

5. Rewrite $A = 0.56t^2 + 100$ in terms of $t$.

$$A - 100 = 0.56t$$

$$t = \left(\frac{A - 100}{0.56}\right)^{\frac{1}{2}}$$
\[ b^3 = 8 \]
\[ \sqrt[3]{b^3} = \sqrt[3]{8} \]
Multiplying and Dividing Rational Expressions

Lesson Target

Rational Expression – Is the quotient of two polynomials
expressions

Examples of Rational Expressions:

\[ \frac{2x^2 + 4x - 3}{2x + 1} \quad \frac{x - 3}{x^2 - 1} \]

Simplified Rational Expression – A rational expression such that the numerator and denominators have no common factors (other than ±1).

Simplify: Like Terms can cancel out.

\[ \frac{x}{x^3} = \frac{1}{x^2} \]

\[ \frac{4x^2 y^3}{8x y^2 z} = \frac{x}{y z} \]

Identical Polynomials can cancel out

\[ \frac{(x - 3)(2x)}{x - 3} = 2x \quad \frac{4x - 8}{4} = x - 2 \]

\[ \frac{(x - 2)(x + 4)}{2(x - 1)(x - 7)} = \frac{1}{2} \]

When Multiplying and Dividing Polynomials: FACTOR AND CANCEL

\[ x^2 + 10x + 25 \div x + 5 \]

\[ \frac{(x + 5)(x + 5)}{(x + 5)} = x + 5 \]

\[ \frac{2x^2 - 50}{x + 5} \]

\[ \frac{2(x - 5)(x + 5)}{x + 5} = 2(x - 5) \]

When dividing KEEP/CHANGE/FLIP

\[ \frac{15}{x - 8} \]

\[ \frac{3}{2x - 16} \]

\[ \frac{15}{x - 8} \div \frac{2x - 12}{3} = \frac{15}{x - 8} \cdot \frac{3}{2(x - 3)} = \frac{45}{2(x - 3)} \]

\[ \frac{32}{5} \]

\[ \frac{x^2 - 9}{x^2 - 6x + 9} \]

\[ \frac{x^2 - 9}{3} \]

\[ \frac{(x - 3)(x + 3)}{3} \cdot \frac{9}{3} = \frac{1}{x^2 - 6x + 9} \]

\[ \frac{3(x + 3)}{x - 3} \]

\[ \frac{x^3 - 27}{x^2 + 3x + 9} \]

\[ \frac{(x - 3)(x^2 + 9)}{x - 3} = x^2 + 3x + 9 \]

\[ \frac{(x + 3)(x + 2)}{x + 5} \]

\[ \frac{(x + 3)(x + 2)}{(x + 5)(x + 1)} \]

\[ \frac{(x + 3)(x + 2)}{(x + 5)(x + 1)} \]

\[ \frac{x + 2}{y + 3} \]

\[ \frac{(x + 3)(x + 2)}{(x + 5)(x + 1)} \]

\[ \frac{x + 2}{y + 3} \]
Simplify:

\[
\frac{x^2 + 2x - 8}{x^2 - 4} \cdot \frac{x^2 - 8x + 16}{x^2 - 16}
\]

\[
\frac{(x+4)(x-2)}{(x-2)(x+2)} \cdot \frac{(x-4)(x+4)}{(x-4)(x+4)}
\]

\[
\frac{x - 4}{x + 2}
\]
### Algebra II Lesson 5.2

#### Adding and Subtracting Rational Expressions

To add or subtract two rational expressions with unlike denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add or subtract.

**Rule:**
1. Find the Common Denominator
2. Combine the Numerator

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{3}{2x} + \frac{3 + 12x}{2x} ]</td>
<td>[ \frac{5d - 3 + 2d - 2}{3d - 1} + \frac{2d}{3d - 1} ]</td>
</tr>
<tr>
<td>[ \frac{3 + 3 + 12x}{2x} ]</td>
<td>[ \frac{5d - 3 + 2d - 2}{3d - 1} ]</td>
</tr>
<tr>
<td>[ \frac{6 + 12x}{2x} ]</td>
<td>[ \frac{-7d - 5}{3d - 1} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{3}{x} + \frac{5x}{x+3} ]</td>
<td>[ \frac{x}{x-1} + \frac{4}{x+5} ]</td>
</tr>
<tr>
<td>[ \frac{3(x+3)}{x(x+3)} + \frac{5x(x)}{x(x+3)} ]</td>
<td>[ \frac{(y-1)(y+5)}{(y-1)(y+5)} ]</td>
</tr>
<tr>
<td>[ \frac{2x}{x+3} ]</td>
<td>[ \frac{(y-1)(y+5)}{(y+5)} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simplify</th>
<th>Simplify</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{5}{x^2-100} + \frac{3}{x+10} ]</td>
<td>[ \frac{3x}{x^2-9} + \frac{2x-3}{(x-3)(x+3)} + \frac{2x-3}{(x+3)(x-3)} ]</td>
</tr>
<tr>
<td>[ \frac{5}{(x-10)(x+10)} + \frac{3(x-10)}{(x+10)(x-10)} ]</td>
<td>[ 3x^2 + 9x + 6x - 3 ]</td>
</tr>
<tr>
<td>[ \frac{5 + 3x - 30}{(x-10)(x+10)} = \frac{3x-25}{(x-10)(x+10)} ]</td>
<td>[ 3x^2 + 11x - 3 ]</td>
</tr>
<tr>
<td>[ \frac{5 + 3x - 30}{(x-10)(x+10)} = \frac{3x-25}{(x-10)(x+10)} ]</td>
<td>[ \frac{3x}{(x+3)(x-3)} ]</td>
</tr>
</tbody>
</table>
Simplify: \( \frac{x}{x-2} - \frac{5}{x+1} \)
\[
\frac{x(x+1)}{(x-2)(x+1)} - 5 \frac{x-2}{(x-2)(x+1)}
\]
\[
x^2 + x - 5x + 10
\]
\[
\frac{x^2 - 4x + 10}{(x-2)(x+1)}
\]

Simplify: \( \frac{3}{x+7} - \frac{2x}{x-1} \)
\[
\frac{3(x-1)}{x+7(x-1)} - 2x \frac{x+7}{x+7(x-1)}
\]
\[
3x - 3 - 2x^2 - 14x
\]
\[
\frac{-2x^2 - 11x - 3}{(x+7)(x-1)}
\]

\( \frac{3}{x+4} - \frac{4}{x-5} \)

\( \frac{x+4}{(x+2)(x-5)} \)

\( x(x-5) \)

Watch for this!!!!!!

\( 25 - x^2 = -(x^2 - 25) \)

\( \frac{-3}{x^2 - 25} - \frac{4}{x-5} \)

\( \frac{-3}{x-5(x+5)} - \frac{4(x+5)}{(x-5)(x+5)} \)

\( \frac{-3 - 4x - 20}{(x-5)(x+5)} \)

\( \frac{-4x - 23}{(x-5)(x+5)} \)

---

Performance of Understanding. If you can do this problem you have a good handle on the lesson.

Describe the error.
\( \frac{x}{x+2} + \frac{4}{x-5} \)

Describe the error.
\( \frac{3+x}{x-1} - \frac{2x+3}{x-1} \)

\( \frac{x+4}{(x+2)(x-5)} \)

\( \frac{6-x}{x-1} \)

\( 3 + x - 2x + 3 \)
### Algebra II Lesson 5.3

#### Solving Rational Equations

**Lesson Target:**

1. I can solve rational equations

**Strategy for solving:**

- Factor denominators if necessary.
- Determine the values that cannot be a solution to the equation.
- Eliminate the denominators.
- Solve the resulting equation.
- Check for extraneous solutions.

If there is one term on each side (Direct Variation) you can cross multiply.

Solve the following:

\[
\frac{3}{x+1} = \frac{9}{4x+5}
\]

**Special Case:** If 1 term each side, cross multiply

**Solve for x:**

\[
\frac{4}{2x} = \frac{5}{x+6}
\]

\[
10x = 4(x+6)
\]

\[
10x = 4x + 24
\]

\[
x = 4
\]

\[
\frac{x-3}{x+5} = \frac{x}{x+2}
\]

\[
x^2 + 5x = x^2 - 3x + 2x - c
\]

\[
x^2 + 5x = x^2 - x - c
\]

\[
5x = -x - c
\]

\[
x = -1
\]

When there is more than one term per side, find the common denominator and solve the numerator.

**What does x equal to?**

\[
\frac{x}{7} + \frac{2}{7} = \frac{5}{7}
\]

\[
x + 2 = 5
\]

\[
x = 3
\]

\[
\frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}
\]

**LCM = 6x**

\[
\frac{2(2)}{2(3x)} + \frac{1}{6} = \frac{4}{3x(2)}
\]

\[
x + 3 = 5
\]

\[
x = 2
\]

\[
\frac{3}{x} + \frac{5c}{7} = \frac{1}{17x}
\]

\[
3x + 5c = 7x
\]

\[
5c = 4x
\]

\[
x = 14
\]
\[
\begin{align*}
\frac{x+1}{x+6} + \frac{1}{x} &= \frac{2x+1}{x+6} \\
\therefore x(x+1) + x+6 &= x(2x+1) \\
x^2 + x + 6 &= 2x^2 + x \\
x^2 + 2x + 6 &= 2x^2 + 7 \\
-x^2 - 2x - 6 &= 0 \\
(x-3)(x+2) &= 0 \\
x = 3, x = -2
\end{align*}
\]

\[
\begin{align*}
\frac{5x}{x-2} &= \frac{7}{x-2} + \frac{10}{x} \\
5x &= 7x - 14 \\
-2y &= -14 \\
x &= 7
\end{align*}
\]

**Performance of Understanding.** If you can do this problem you have a good handle on the lesson.

**Find the errors.**

\[
\begin{align*}
\frac{3}{2x} + \frac{4}{x^2} &= 1 \\
3x^2 + 8x &= 1
\end{align*}
\]

\[
\begin{align*}
5 + \frac{23}{x} &= \frac{45}{x} \\
\frac{28}{x+6} &= \frac{45}{x}
\end{align*}
\]
Horizontal asymptote
- Give \( y = \frac{a(x)}{b(x)} \)
- \( y \) has at most one horizontal asymptote.

Case 1: If the degree of \( a(x) \) is greater than the degree of \( b(x) \), there is no horizontal asymptote \( a(x) > b(x) \)
Case 2: If the degree of \( a(x) \) is less than the degree of \( b(x) \), the horizontal asymptote is the line \( y = 0 \). \( a(x) < b(x) \); \( y = 0 \)

Graph: \( y = \frac{2x+3}{x+1} \)

- \( x \)-intercepts \(-3\frac{1}{2}, 0\)
- \( y \)-intercepts \(0, 3\)
- Vertical asymptotes \( x = -1 \)
- Horizontal asymptotes \( y = 2 \)

Case 3: If the degree of \( a(x) \) equals the degree of \( b(x) \), the horizontal asymptote is the line
\[
y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}
\]
- If the factor of the numerator and the denominator match, there is a hole.

Sketch the graph

Performance of Understanding: If you understand this problem you understand the lesson.

Graph \( y = \frac{x+3}{x^2+x-6} \)

\[
y = \frac{x+3}{(x+3)(x-2)} = \frac{1}{x-2}
\]

\( x + 3 = 0 \)
\( x = -3 \)

HA \( y = 0 \)
VA \( x = 2 \)

\( y = \frac{1}{-3-2} = -\frac{1}{5} \)

hole \((-3, -\frac{1}{5})\)

\( y_{1n+} = \frac{1}{0-2} = -\frac{1}{2} \)

\( x_{1n+} \) none
### Graphing Rational Equations

**Parent equation:** \( y = \frac{1}{x} \)

**Shape:** Hyperbola

**Asymptotes:** \( x = 0 \) and \( y = 0 \)

Not defined at \( x = 0 \)

The form of the equation is:

\[
\frac{y}{x-h} + k = \frac{1}{x}
\]

- **Vertical Asymptote:** \( x = h \)
- **Horizontal Asymptote:** \( y = k \)

---

#### Describe the shift of the Rational Functions

1. \( y = \frac{4}{x-4} + 2 \)
   - **Vertical Asymptote:** \( x = 4 \)
   - **Horizontal Asymptote:** \( y = 2 \)
   - **Intersection Points:**
     - \( x_{int} = (2, 0) \)
     - \( y_{int} = (0, 1) \)
   - **Graph:**

2. \( y = \frac{-3}{x+1} - 4 \)
   - **Vertical Asymptote:** \( x = -1 \)
   - **Horizontal Asymptote:** \( y = -4 \)
   - **Intersection Points:**
     - \( x_{int} = (-2, 0) \)
     - \( y_{int} = (-4, -4) \)
   - **Graph:**

3. \( y = \frac{-3}{x+3} \)
   - **Vertical Asymptote:** \( x = -3 \)
   - **Horizontal Asymptote:** \( y = 0 \)
   - **Intersection Points:**
     - \( x_{int} = (none) \)
     - \( y_{int} = (0, -3) \)
   - **Graph:**
### Algebra II Lesson 6.1

#### Function Notation

$f(x)$ replaces $y$

Function: the value of $x$ does not repeat

Vertical Line Test (VLT): If a Vertical Line passes through a relation only once – it is a function.

<table>
<thead>
<tr>
<th>$x = 2$</th>
<th>$y = 3$</th>
</tr>
</thead>
</table>

Let $f(x) = 3x^2 - 9$

Find the value of the function at $x = 2$

Replace $x$ with 2

$f(2) = 3(2)^2 - 9$

$f(2) = 3(4) - 9$

$f(2) = 12 - 9$

$f(2) = 3$

When $x$ is 2 the value of the function is 3

**Domain:** $\mathbb{R}$  
**Range:** $y = -9$

---

**Evaluating functions**

Let $f(x) = 6x^2 + 4x^2 - 48x$

Find $f(-1)$

\[
(f(-1))^3 + 4(f(-1))^2 - 4f(-1)
\]

\[
= -6 + 4 + 48 = 46
\]

**Domain:** $\mathbb{R}$  
**Range:** $\mathbb{R}$

---

**Finding the value of $x$ algebraically**

Let $f(x) = |x - 3| + 2$

Find the value(s) of $x$ if $f(x) = 3$

\[
3 = |x - 3| + 2
\]

\[
+1 = |x - 3|
\]

\[
x - 3 = 1 \quad x - 3 = -1
\]

\[
x = 4 \quad x = 2
\]

**Domain:** $\mathbb{R}$  
**Range:** $y \geq 2$

---

**Finding the value of $x$ algebraically**

Let $f(x) = (x - 1)^2 + 2$

Find the value(s) of $x$ if $f(x) = 4$

\[
(x - 1)^2 = 4
\]

\[
(x - 1) = \pm 2
\]

\[
x - 1 = 4 \quad x = 5
\]

**Domain:**  
**Range:**
### Operations on functions

Let \( f(x) = 1 - x^2 \) and \( g(x) = x + 1 \)

Find \( f(x) - g(x) \)

\[
f(x) - g(x) = (1 - x^2) - (x + 1)
\]

\[
f(x) - g(x) = 1 - x^2 - x - 1
\]

\[
f(x) - g(x) = -x^2 - x
\]

---

### Operations on functions

Let \( f(x) = x^2 - 1 \) and \( g(x) = x + 1 \)

Find \( f(x) \div g(x) \)

\[
\frac{x^2 - 1}{x + 1} = \frac{(x-1)(x+1)}{x+1} = x - 1
\]

---

### I can Evaluate Functions Graphically

**Evaluate Graphically**

\( f(-1) = \)

\( f(-1) = -2 \)

Domain: \( \mathbb{R} \)

Range:

\( y \geq -3 \)

---

### I can Evaluate Functions Graphically

**Evaluate Graphically**

\( f(0) = \)

\( f(0) = -3 \)

\( f(1) = \)

\( f(1) = -2 \)

\( f(-2) = \)

\( f(-2) = 1 \)

---

### Solve Graphically

**Solve Graphically**

Find \( x \) when \( f(x) = 0 \)

\( f(x) = 0 \) when

\( x = 2 \) or \( x = -2 \)

Domain: \( \mathbb{R} \)

Range:

\( y \geq -4 \)

---

### Solve Graphically

**Solve Graphically**

Find \( x \) when \( f(x) = -3 \)

\( x = 1 \uparrow -1 \)

Find \( x \) when \( f(x) = -4 \)

\( x = 0 \)

---

### Performance of Understanding

**If you can do this you have a good understanding of the lesson**

\( f(x) = x^3 - x + 1 \)

Find: \( f(x - 1) \)

\[
(f(x - 1))^2 - (x - 1) + 1 = (x^2 - 2x + 1 - x + 1 + 1)
\]

\[
x^2 - 3x + 3
\]
Algebra II

Example
Increasing: $\infty \leq x \leq 3$

Decreasing: $-\infty \leq x \leq -3$

As $x \to \infty$, $y \to \infty$
As $x \to -\infty$, $y \to -\infty$

Example
Increasing: $-\infty \leq x \leq 3$

Decreasing: $3 \leq x \leq \infty$

As $x \to \infty$, $y \to -\infty$
As $x \to -\infty$, $y \to -\infty$

Example
Increasing: $(-\infty, -2)$, $(3, \infty)$

Decreasing: $(-2, 3)$

Performance of Understanding
If you can do this problem you have a good understating of the lesson!!!!

The graph below represents $h(x)$

Increasing: none
Decreasing: $-\infty \leq x \leq 5$, $5 \leq x \leq \infty$

Find:
$h(0) = 0$, $h(1) = 0$

Find $x$ when $h(x) = 5$
Definitions

A zero or root of a function is a value of x that makes $f(x) = 0$.

A local/relative maximum is the value of $f(x)$ that is more than all nearby $f(x)$ values.

A local/relative minimum is the value of $f(x)$ that is less than all nearby $f(x)$ values.

Zeros: $-2, -1, 0, \frac{2}{3}$

$(x+2)(x+1)(x-1)$

Relative maximum: $-\frac{4}{9}, 15c$

Relative minimum: $(-1, -2, -1), (0, 2, 1)$

Domain: $(-\infty, \infty)$ Range: $[-1, \infty)$

Example

Relative maxima: none

Relative minima: $-3, -1$

Zeros: $-2, -4$

As $x \to \infty, y \to \infty$ As $x \to -\infty, y \to \infty$

Domain: $\mathbb{R}$ Range: $y \geq -1$

Definitions

A graph is increasing on an open interval of x if the y values are increasing as x increases.

A graph is decreasing on an open interval of x if y values are decreasing as x increases.

Example

Increasing: $-3, 1, y = -0.5c$

Decreasing: $-3.14 < x < -1.05c$

Domain: $\mathbb{R}$ Range: $y \geq -1$

As $x \to \infty, y \to ___$ As $x \to -\infty, y \to ___$

Example

Relative maxima: $(-4, 74, 126)$

Relative minima: $(-6.3, -2.07)$

Zeros: $-6, -3, 1$

$(x+c)(x+y+3x)(x-1)$

As $x \to \infty, y \to ___$ As $x \to -\infty, y \to ___$

Domain: $\mathbb{R}$ Range: $\mathbb{R}$
**Composition Functions**

**Definition**

The composition of a function \( g \) with a function \( f \) is : \( f(g(x)) \)

To find the composition of two a function \( g \) with a function \( f \) simply replace all the "x's" in \( g(x) \) with the function \( f(x) \).

**Examples**

Given: \( f(x) = 4x \), \( g(x) = 5x - 2 \), \( h(x) = x^2 - 2x \)

Find \( f(g(x)) \)

\[ = f(5x - 2) \]
\[ = 4(5x - 2) \]
\[ = 20x - 8 \]

Find \( f(f(x)) \)

\[ f(x) = 4(x) \]
\[ f(x) = 4(4x) = 16x \]

Find \( h(f(13)) \)

Frist method

\[ h(f(x)) = h(\sqrt[3]{x - 5}) \]
\[ = \sqrt[3]{x - 5} + 8 \]
\[ h(f(13)) = \frac{\sqrt[3]{13 - 5}}{2} + 8 \]
\[ = 10 \]

Second method

\[ h(f(13)) = f(13) = \frac{\sqrt[3]{13 - 5}}{2} \]
\[ = 2 \]
\[ h(f(13)) = h(2) \]
\[ = 10 \]

Given \( f(x) = \sqrt[3]{x - 5} \), \( g(x) = -x^2 \), \( h(x) = x + 8 \)

Find \( g(-3) \)

\[ g(-3) = -9 \]

Find \( f(-9) \)

\[ f(-9) = 3(-9) + 2 \]
\[ = -25 \]

Find \( g(f(\phi)) \)

\[ f(\phi) = \sqrt[3]{-1} = -1 \]

\[ g(-1) = -1 \]

**Inverse Functions**

**Definition**

An inverse relation interchanges the input and output values of the original relation.

Functions \( f \) and \( g \) are inverses of each other if:

\[ f(g(x)) = x \quad \text{and} \quad g(f(x)) = x \]

The function \( g \) is denoted by \( f^{-1} \), read as "\( f \) inverse."

**Verify that \( f(x) = 3x - 5 \) and \( f^{-1}(x) = \frac{1}{3} x + \frac{5}{3} \) are inverse functions.**

Show \( f(f^{-1}(x)) = x \)

\[ f\left(\frac{1}{3} x + \frac{5}{3}\right) \]
\[ = 3\left(\frac{1}{3} x + \frac{5}{3}\right) - 5 \]
\[ = x + 5 - 5 \]
\[ = x \]
**Finding the inverse of a function**

Replace \( f(x) \) with \( y \)

Solve for \( y \)
Replace \( y \) with \( f^{-1}(x) \)

**Find the inverse of \( f(x) \)**

\[
f(x) = 2x + 1
\]

\[
y = 2x + 1
\]

\[
x = 2y + 1
\]

\[
x - 1 = 2y
\]

\[
\frac{x - 1}{2} = y
\]

**Inverse relationship**

Switch the inputs and the outputs
Switch the "x" and the "y"

**Give the graph of the inverse relationship**

Performance of Understanding

If you can do this problem you have a good understanding of the whole lesson

Given: \( f(x) = y, \quad g(x) = 2x^2, \quad h(x) = 3x^3 \)

Find: \( h(g(f(x))) = \)
Characteristics of a system of equations consists of two or more equations and two or more unknowns.

Example:

\[
\begin{align*}
(x^2 - 3y &= 0) \\
y &= 2x + 3 
\end{align*}
\]

A solution to a system of equations of two variables is an ordered pair that satisfies all of the equations in the system.

Solve the following system of equations:

\[
\begin{align*}
y &= 2x^2 - 7 \\
y &= x^2 - 3 \\
\text{Work:} & \quad 2x^2 - 7 = x^2 - 3 \\
x^2 - 4 &= 0 \\
x^2 &= 4 \\
x &= \pm 2 \\
y &= (2)^2 - 3 \\
y &= 4 - 3 \\
y &= 1 \\
y &= (-2)^2 - 3 \\
y &= 4 - 3 \\
y &= 1 \\
(2, 1) \\
(-2, 1) 
\end{align*}
\]

Example of a solution to a system:

A solution to the system \(y = x + 4\) and \(y = 2x - 8\) is \((12, 16)\).

This ordered pair must satisfy both equations. Let's show that!!

\[
\begin{align*}
y &= x + 4 \\
y &= 2x - 8 \\
16 &= 12 + 4 \\
16 &= 2(12) - 8 \\
16 &= 16 \\
16 &= 16 
\end{align*}
\]

Solve the following system of equations:

\[
\begin{align*}
y &= x^2 - 4x + 6 \\
+ y &= 6 \\
x^2 - 4x + c &= -x + 6 \\
+ x \\
x^2 - 3x + c &= c \\
-x - c \\
x^2 - 3x = 0 \\
x(x - 3) &= 0 \\
x &= 0 \\
x &= 3 \\
y &= x - c \\
y &= -3 + c \\
(0, 6) \\
(3, 3) 
\end{align*}
\]
Solve the following system of equations

\[-2x + y = -6\]
\[y = -2x^2 + 16x - 26\]

\[-2x^2 + 16x - 26 = 2x - 6\]
\[-2x + 6\]
\[-2 \cdot 2 + 14 \cdot 1.5 - 20 = 0\]
\[-2 \cdot (x^2 - 7x + 10) = 0\]
\[(x - 2)(x - 5) = 0\]
\[x = 2, x = 5\]
\[y = 2(2) - 6, y = 2(5) - 6\]
\[y = -2, y = 4\]
\[2, -2, 5, 4\]

Solve the system

\[y = x^2 - 4x + 3\]
\[2x + y = 3\]

\[y = -2x + 3\]

Performance of Understanding
If you can solve this you have a good understanding of the material

Solve the system using a graphing calculator

\[(2, -1)\]
\[(0, 3)\]
Finding Intervals of Increase and Decrease Using TI-84

1. Type \( y = x^3 + 7x + 3 \) into \( y = \) and hit the graph button.
2. Sketch the graph at right.
3. Find the max.
   - \( 2^{nd} \), trace, 4, left bound “enter”, right bound “enter”, guess “enter”
   - Label the max on the graph. Underline the \( x \) value.
4. Find the Min.
   - \( 2^{nd} \), trace, 3, left bound “enter”, right bound “enter”, guess “enter”
   - Label the min on the graph. Underline the \( x \) value.
5. Draw the arrows and right down the interval.

Finding zeroes Using TI-84

1. Type \( y = 0 \) into \( Y2 = \)
2. The intersection of \( y = x^3 + 7x + 3 \) and \( y = 0 \) are the zeros.
3. To find the intersection: \( 2^{nd} \), trace 5: Intersect, enter, enter, enter. That will find the first zero.
4. To find the others, you must place the flashing “X” over the zero before you hit enter, enter.

Find the Domain and Range of \( y = (x - 3)^2 + 2 \)

1. Sketch the graph at right.

   Domain \( (-\infty, \infty) \)

   Range \( (3, \infty) \)

Find the Domain of \( h(x) = \frac{x^2}{x-2} \)

\[ x \neq 2 \]

\[ m = \lim_{x \to 2} \frac{16}{4-2} = \frac{16}{2} = 8 \]

Real zeroes or not: (put into graphing calc)

1. \( f(x) = -(x - 1)^2 - 3 \)

   \( y = 3|x + 2| - 4 \)

2. \( g(x) = \sqrt{x - 2} - 7 \)

   \( h(x) = \frac{x+4}{x-1} \)
Write a rule for the nth term of the sequence.

\[2, 6, 18, 54 \ldots\]

1st find \( a_1 \) and \( r \)

\[ r = \frac{6}{2} = 3 \]

2nd Substitute \( a_1 \) and \( r \) into \( a_n = a_1 r^{n-1} \)

\[ a_n = 2(3)^{n-1} \]

Find the 8th term in the sequence.

\[ a_8 = 2(3)^7 = 4374 \]

Write a rule for the sequence and find \( a_{23} \).

\[ \{12, 18, 27, \frac{81}{2} \ldots\} \]

\[ a_n = 12 \left( \frac{1.5}{2} \right)^{n-1} \]

\[ a_{23} = 12 \left( \frac{1.5}{2} \right)^{22} = 89.7419371 \]

One term of a geometric sequence is \( a_5 = 81 \) and the common ratio is 3.

Write a rule for the nth term.

\[ a_1 = \frac{1}{1-\frac{3}{2}} = \frac{2}{3} \]

\[ a_3 = \frac{3}{3+1} = \frac{3}{4} \]

\[ a_2 = \frac{2}{2+1} = \frac{2}{3} \]

\[ a_n = \frac{n}{n+1} \]

Write the terms of a sequence.

Find the first 4 terms of the sequence. Round to two decimal places.

\[ a_n = \frac{\sqrt{n} + 3}{3 + \sqrt{n}} \]

\[ a_1 = 1 \]

\[ a_2 = 1.0362 \]

\[ a_3 = 1.0652 \]

\[ a_4 = 1.0899 \]
**Definition:** An **arithmetic sequence** is a sequence in which each term after the first is found by adding a constant, called the **common difference**, \( d \), to the previous term.

**Example 1:** Fill in the next 3 terms. What is the common difference?

\[
2, 5, 8, 11, 14, 17, 20, 23, 26, \ldots
\]

\[
d = 3
\]

\[
a_1 = 2
\]

**Example 2:** Fill in the next 3 terms. What is the common difference?

\[
55, 49, 43, 37, 31, 25, \ldots
\]

\[
d = -6
\]

\[
a_1 = 55
\]

**Formula for writing an equation of an arithmetic sequence:**

The \( n \)th term \( a_n \) of an arithmetic sequence with first term \( a_1 \) and a common difference \( d \) is given by:

\[
a_n = a_1 + (n-1)d
\]

Where \( n \) is any positive integer (\( n \) is the number of terms in the sequence)

**Write an equation (rule) for the nth term of the arithmetic sequence 8, 17, 26, 35, ...**

1\(^{st}\) find \( a_1 \) and \( d \)

\[
a_1 = 8 \quad \text{and} \quad d = 9
\]

2\(^{nd}\) Substitute \( a_1 \) and \( d \) into \( a_n = a_1 + (n-1)d \)

\[
a_n = 8 + (n-1)9
\]

3\(^{rd}\) simplify

\[
a_n = 8 + 9n - 9
\]

\[
a_n = 9n - 1
\]

**Find the 15\(^{th}\) term in the sequence?**

\[
9(15) - 1 = 134
\]

**I can write a Geometric Sequence**

**Definition:** In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by \( r \).

**Example 1:** Fill in the next 3 terms. What is the common ratio?

\[
3, 6, 12, 24, 48, 96, 192
\]

\[
r = \frac{6}{3} = 2
\]

\[
a_3 = 2
\]

**Example 2:** Fill in the next 3 terms. What is the common ratio?

\[
6 \frac{1}{4}, 32, 1 \frac{1}{2}, \frac{1}{2}, \frac{7}{8}, \frac{3}{4}
\]

\[
r = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{1}{2}
\]

\[
a_1 = 6 \frac{1}{4}
\]

**Formula for writing an equation of a geometric sequence:**

The \( n \)th term of a geometric sequence with first term \( a_1 \) and common ration \( r \) is given by:

\[
a_n = a_1 r^{n-1}
\]

Where \( r = \frac{a_2}{a_1} \)
When the terms of a sequence are added together, the resulting expression is a **series**. A series can be finite or infinite.

**Finite series:** \( 2 + 4 + 6 + 8 \)

**Infinite series:** \( 2 + 4 + 6 + 8 + \ldots \)

When the terms of a sequence are added together, the resulting expression is a **series**. A series can be finite or infinite.

\[ s_n = \frac{n}{2} [2a_1 + (n-1)d] \]

Where \( n \) is any positive integer \( (n \) is the number of terms in the sequence)

\( d \) = common difference

\( a_1 \) = first term in the sequence

---

Find the sum of the first 102 terms of the series \( \{-6, -4.5, -3, -1.5, \ldots\} \)

1st find \( a_1, n \) and \( d \)

\[ a_1 = -6 \quad n = 102 \quad d = -1.5 \]

2nd Substitute \( a_1, n \) and \( d \) into \( s_n = \frac{n}{2} [2a_1 + (n-1)d] \)

\[ s_{102} = \frac{102}{2} [2(-6) + (102-1)(-1.5)] \]

3rd solve: \( s_{102} = 51 (-12 + 101 (1.5)) \)

\[ s_{102} = 7114.5 \]

---

I can find the sum of a **Geometric Series**

The sum of a finite geometric series

The sum of the first \( n \) terms of a geometric series with common ratio \( r \neq 1 \) is:

\[ s_n = a_1 \left( \frac{1 - r^n}{1 - r} \right) \]

---

Find the sum of the first 12 terms of the sequence \( 5, 15, 45, \ldots \)

\[ s_{12} = 5 \left( \frac{1 - 3^{12}}{1 - 3} \right) \]

\[ s_{12} = 132,960 \]

---

Find \( S_8 \) algebraically if the series is \( 100, 50, 25 \ldots \)

\[ S_8 = 100 + \frac{1 - \left(\frac{1}{2}\right)^8}{1 - \frac{1}{2}} \]

\[ S_8 = 199.2 \]
Lesson 7.2

Bill runs every day. He increases the amount of time he runs every day by 3 minutes. On his first day of running he spends 30 minutes running. After his 12th day, how many total minutes has Bill run?

I can find the sum of an infinite Geometric Series

The sum of an infinite geometric series with the first term $a_1$ and common ratio $r$ is given by:

$$s_\infty = \frac{a_1}{1-r} \quad \text{Given: } |r| < 1$$

If $|r| \geq 1$ the series has no sum.

Note: $r$ is a fraction between -1 and 1

Find $s_\infty$ for the following series

$$3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \ldots$$

Is this an infinite series? Yes/No

$$\frac{3}{4} = \frac{1}{4}$$

Is $r$ a fraction between -1 and 1? Yes/No

$$s_\infty = \frac{3}{1-\frac{1}{4}} = \frac{3}{\frac{3}{4}} = 4$$

Find $s_\infty$ for the following series

$$6 + \frac{3}{2} + \frac{3}{4} + \ldots$$

$$r = \frac{1}{2}$$

$$\frac{3}{1-\frac{1}{2}} = 6$$

Find $s_\infty$ for the following series

$$3 + \frac{15}{4} + \frac{75}{16} + \ldots$$

$a_1 = 3$ $r = \frac{5}{4}$

$$s_\infty = \frac{a_1}{1-r} = \frac{3}{1-\frac{5}{4}} = -12$$

Performance of Understanding:

Find the error:

Find $s_\infty$ of the following series
<table>
<thead>
<tr>
<th>Recursive rule for arithmetic and geometric sequences</th>
<th>Arithmetic sequence</th>
<th>Geometric Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Explicit Rule</strong></td>
<td>$a_n = a_1 + (n - 1)d$</td>
<td>$a_n = a_1 r^{n-1}$</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td>$s_n = \frac{n}{2}[2a_1 + (n - 1)d]$</td>
<td>$s_n = a_1 \frac{1 - r^n}{1 - r}$</td>
</tr>
<tr>
<td><strong>Sum of Infinite Series</strong></td>
<td>None</td>
<td>$s_\infty = \frac{a_1}{1 - r}, -1 &lt; r &lt; 1$</td>
</tr>
<tr>
<td><strong>Recursive Rule</strong></td>
<td>$a_n = a_{n-1} + d$</td>
<td>$a_n = r \cdot a_{n-1}$</td>
</tr>
</tbody>
</table>

Write a recursive rule for the sequence 3, 13, 23, 33, 43... \( \bigcirc \) or \( \bigcirc \)

- \( d = 10 \)
- \( a_n = a_{n-1} + 10 \)

Write a recursive rule for the sequence 16, 40, 100, 250,... \( A \) or \( \bigcirc \)

- \( r = \frac{5}{2} \)
- \( a_n = a_{n-1} \cdot \left(\frac{5}{2}\right) \)

Write a recursive rule for the sequence 16, 8, 4,... \( A \) or \( \bigcirc \)

- \( r = \frac{1}{2} \)
- \( a_n = a_{n-1} \cdot \frac{1}{2} \)

Write a recursive rule for the sequence 23, 20, 17, 14,... \( A \) or \( \bigcirc \)

- \( d = -3 \)
- \( a_n = a_{n-1} - 3 \)

Give the first four terms of the sequence:
- \( a_1 = 12 \)
- \( a_n = 3a_{n-1} + 2 \)

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>38</td>
<td>110</td>
<td>330</td>
</tr>
</tbody>
</table>

Give the first four terms of the sequence:
- \( a_1 = -3 \)
- \( a_n = 2 \cdot a_{n-1} \)

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-6</td>
<td>-12</td>
<td>-24</td>
</tr>
</tbody>
</table>
I can use Summation Notation

\[ \sum_{i=1}^{4} 2i \]

Index: for this example is \( i \)
Lower limit of summation: 1
Upper limit of summation: 4
\( \Sigma \) is called sigma. This notation is sometimes called sigma notation.

\[ \sum_{i=1}^{4} 2i = 2(1) + 2(2) + 2(3) + 2(4) \]

\[ \sum_{k=1}^{3} k^2 + 1 = 18 \]

\( k = 1 \)
\[ 1^2 + 1 = 3 \]

\( k = 2 \)
\[ 2^2 + 1 = 5 \]

\( k = 3 \)
\[ 3^2 + 1 = 10 \]

\[ \frac{10}{18} \]

Write the series using summation notation:

\[ \frac{16 + 8 + 4 \ldots}{2 + 3 + 4 \ldots 2(2)} \]

1. Find the Rule.
\[ a_n = 16 \left( \frac{1}{2} \right)^{n-1} \]

2. Write the Sigma Notation

\[ \sum_{n=1}^{\infty} \frac{1}{2^n} \]

Write the series using summation notation:

\[ 5 + 10 + 15 + 20 + 25 \]

\[ a_n = 5 + (n-1)5 \]

\[ a_n = 5 + 5n - 5 = 5n \]

\[ \sum_{n=1}^{5} 5n \]

POU: Describe and correct the error in finding the sum of the series

\[ \sum_{i=0}^{3} (2i + 3) = 5 + 7 + 9 + 11 + 13 = 45 \]